Supplemental Appendix for
”Robust Estimation for Average Treatment Effects”

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Part II
All Simulation Results

Appendix H contains tables with all simulation results. Tables H.1-H.4 in Appendix H.1 contain all estimation results in which the covariate X is scalar, Z has a *symmetric* distribution and the *true* propensity score is used for infeasible ATE estimation. Tables H.5- H.8 in Appendix H.2 contain results in which X is scalar, Z has an *asymmetric* distribution and the *true* propensity score is used. Tables H.9-H.24 in Appendices H.3-H.6 contain results for each simulation in which an *estimated* propensity is used.

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H Simulation Results: All Tables

Tables H.1-H.4 in Section H.1 contain all estimation results in which the covariate X is scalar, Z has a *symmetric* distribution and the *true* propensity score is used for infeasible ATE estimation. Tables H.5- H.8 in Section H.2 contain results in which X is scalar, Z has an *asymmetric* distribution and the *true* propensity score is used. Tables H.9-H.24 in Sections H.3-H.6 contain results for each simulation in which an *estimated* propensity is used, denoted as Cases 1-4, covering scalar and multivariate X, and symmetrically and asymmetrically distributed Z. The four cases are as follows, where $\beta \in \{.25, 1, 2\}$, and $\tilde{X}_i$ are stochastic covariates.

Case 1. The data are generated with a scalar covariate $X_i = \tilde{X}_i$, and $\gamma = \beta$ and $(Y_{0,i}, Y_{1,i}, \tilde{X}_i, U_i)$ have the various distributions described in the main simulation section. We include a constant term for estimation, hence $[1, \tilde{X}_i]$ is used for estimating $\gamma_0 = [0, \beta]'$.

Case 2. This repeats Case 1, except we add and estimate a constant term. The data are generated with covariates $X_i = [1, \tilde{X}_i]$ for scalar $\tilde{X}_i$; $\gamma_0 = [.25, \beta]$; $(Y_{0,i}, Y_{1,i}, \tilde{X}_i, U_i)$ are as above; and $[1, \tilde{X}_i]$ is used for estimating γ_0.

The last two cases have multiple stochastic covariates.

Case 3. We now use multiple stochastic covariates $\tilde{X}_i = [\tilde{X}_{j,i}]_{j=1}^3$, $\tilde{X}_{1,i}$ is Bernoulli with $P(X_{1,i} = 1) = .3$, $\tilde{X}_{3,i} = \tilde{X}_{2,i}^2$, and $(Y_{0,i}, Y_{1,i}, \tilde{X}_{2,i}, U_i)$ are as above; $\gamma_{0,1} = .5$, $\gamma_{0,2} = \beta$ and $\gamma_{0,3} = \beta/2$. We include a constant term for estimating $\gamma_0 = [0, .5, \beta, \beta/2]'$.

Case 4. This repeats Case 3 except we add and estimate a constant term. The covariates are $\tilde{X}_i = [\tilde{X}_{j,i}]_{j=1}^4$, $\tilde{X}_{1,i} = 1$, $\tilde{X}_{2,i}$ is Bernoulli with $P(X_{2,i} = 1) = .3$, $\tilde{X}_{4,i} = \tilde{X}_{3,i}^2$, and $(Y_{0,i}, Y_{1,i}, \tilde{X}_{3,i}, U_i)$ are as above; the constant term is $\gamma_{0,1} = .25$, and remaining parameters are $(\gamma_{0,2}, \gamma_{0,3}, \gamma_{0,4}) = (.5, \beta, \beta/2)$.

H.1 One Covariate, Known $p(X)$, and Symmetric Z

Table H.1. (a) Estimator Properties (Symmetric Z, known $p(X)$, Normal *or* Laplace, $n = 100$, 250)

	$n=100$									$n=250$									
		$(Y_0,Y_1,X,U)\sim$ **Normal**				$(Y_0,Y_1,X,U)\sim$ **Laplace**					$(Y_0,Y_1,X,U)\sim$ **Normal**				$(Y_0,Y_1,X,U)\sim$ **Laplace**			
		$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$					$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0023	.0025	.2027	.6031	.0018	.0020	.2179	.5773	0	-.0004	-.0006	.1289	.4570	.0015	.0018	.1366	.4855
TT(Z)	1	.0013	-.0002	2058	.5469	.0012	.0003	.2145	.5760	.4	-.0007	-.0006	.1295	.6493	.0010	.0001	.1341	.5245
TT-BC(Z)	1	.0013	.0001	.2055	.4101	.0013	.0004	.2129	.8190	.4	-.0007	-.0006	.1294	.4697	.0010	.0005	.1332	.7832
TT(X)	13	.0021	.0017	.1989	.5868	.0012	.0024	.2068	.6970	8.7	-.0005	-.0011	.1275	.5491	.0021	.0037	.1309	.7071
TT(X,$k_n^{(x)}$)	43	.0020	.0019	.1513	.4316	-.0005	.0013	.1826	.4713	36	-.0002	.0002	.1003	.3879	.0010	.0022	.1190	.6126
TT(X,k_n)	1	.0026	.0024	.2014	.5039	-.0010	-.0018	.6870	.4945	.4	-.0004	-.0003	.1286	.5406	.0023	.0023	.1363	.4842
TT(Y)	1	.0060	.0082	.2061	.4500	.0064	-.0078	.2357	.8615	.4	.0019	.0071	.1267	.5243	.0013	.0006	.1397	.4245
		$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$					$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0071	.0047	.3376	5.751	.0013	.0017	.4556	8.912	0	.0001	-.0021	.2302	5.632	-.0041	-.0036	.3351	10.21
TT(Z)	1	.0038	.0032	.2126	.9237	.0022	.0043	.2387	1.209	.4	-.0002	-.0035	.1486	1.027	-.0029	-.0016	.1659	1.242
TT-BC(Z)	1	.0037	.0032	.2102	.5484	.0028	.0042	.2389	.6935	.4	-.0002	-.0034	.1469	.9469	-.0029	-.0017	.1622	.6239
TT(X)	13	.0042	.0046	.2809	2.211	.0052	.0051	.2837	.1551	8.7	-.0008	-.0018	.1900	2.159	-.0020	-.0030	.1854	1.246
TT(X,$k_n^{(x)}$)	43	.0023	-.0002	.1602	.6443	.0006	.0012	.1980	.8040	36	.0005	.0007	.1103	.5807	-.0017	-.0009	.1321	.7328
TT(X,k_n)	1	.0049	.0049	.3185	4.505	-.0055	-.0015	.4284	7.408	.4	-.0006	-.0022	.2158	4.424	-.0033	-.0021	.2960	7.317
TT(Y)	1	-.0166	-.0143	.3115	1.006	.0065	.0058	.4053	1.906	.4	-.0229	.0048	.6458	8.351	-.0025	-.0018	.3197	2.886
		$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$					$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0001	-.0010	.6623	16.54	-.0014	.0053	.7859	16.06	0	.0097	.0009	1.137	27.47	-.0021	-.0028	.7826	19.56
TT(Z)	1	-.0008	-.0018	.2063	2.382	.0023	.0013	.2514	2.062	.4	.0006	.0009	.1722	2.143	.0002	.0004	.1946	1.732
TT-BC(Z)	1	.0006	-.0015	.2474	1.425	.0009	.0013	.3012	1.352	.4	.0016	.0007	.2417	1.324	-.0014	.0002	.2409	1.232
TT(X)	13	.0001	-.0008	.6621	16.53	.0059	.0035	.5513	9.964	8.7	.0096	.0010	1.137	27.47	-.0025	-.0025	.3910	8.286
TT(X,$k_n^{(x)}$)	43	-.0008	-.0001	.2034	1.634	.0012	.0019	.2431	1.219	36	.0030	.0016	.1506	1.413	-.0027	-,0019	.1693	1.322
TT(X,k_n)	1	.0002	-.0006	.6623	16.54	.0022	.0005	.7200	13.85	.4	-.0025	-.0033	.7877	27.77	-.0102	-.0033	.6726	18.05
TT(Y)	1	.0366	-.0010	1.048	8.056	.0250	-.0027	.6488	3.909	.4	.0191	.0020	.6472	6.459	-.0116	.0122	.5812	5.217

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.1. (b) Estimator Properties (Symmetric Z, known $p(X)$, Normal *and* Laplace, $n = 100$, 250)

	$n=100$										$n=250$								
		$(Y_0,Y_1,X)\sim$ **Norm,** $U\sim$ **Lap**				$(Y_0,Y_1,X)\sim$ **Lap,** $U\sim$ **Norm**					$(Y_0,Y_1,X)\sim$ **Norm,** $U\sim$ **Lap**				$(Y_0,Y_1,X)\sim$ **Lap,** $U\sim$ **Norm**			
		$\beta=.25$				$\beta=.25$					$\beta=.25$				$\beta=.25$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0005	.0003	.2054	.7790	.0001	.0031	.2189	.7417	0	-.0002	-.0009	.1296	.5263	.0015	.0017	.1388	.5094
TT(Z)	1	.0001	.0010	.2068	.5409	-.0013	-.0007	.2099	.8640	.4	-.0003	-.0002	.1299	.7953	.0022	.0027	..1907	.4206
TT-BC(Z)	1	.0002	.0009	.2066	.6817	-.0013	.0000	.2086	.9786	.4	-.0003	-.0003	.1296	.4572	.0023	.0026	.1315	.6002
TT(X)	13	.0007	-.0005	.2009	.8564	-.0012	.0004	.2032	.8950	8.7	-.0002	.0004	.1283	.7471	.0018	.0029	.1209	.7311
TT(X,$k_n^{(x)}$)	43	-.0003	.0001	.1524	.5368	-.0004	.0004	.1804	.7685	36	.0005	.0001	.1017	.5726	.0008	.0019	.1183	.3993
TT(X,k_n)	1	-.0014	-.0025	.2039	.6515	-.0029	-.0007	.2165	.5335	.4	-.0005	-.0009	.1302	.6876	.0019	.0021	.1368	.4415
TT(Y)	1	.0057	.0019	.2085	.6133	.0004	.0030	.2357	.9575	.4	.0028	.0035	.1282	.5046	-.0007	-.0026	.1433	.5514
		$\beta=1$				$\beta=1$					$\beta=1$				$\beta=1$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0019	-.0031	.2637	1.190	-.0041	-.0050	.5865	14.20	0	-.0035	-.0068	.1164	.8191	-.0108	-.0045	.5499	18.24
TT(Z)	1	-.0025	-.0036	.2179	.6263	.0010	.0030	.2288	1.565	.4	-.0036	-.0051	.1461	.7105	-.0026	-.0027	.1659	1.631
TT-BC(Z)	1	-.0025	-.0035	.2151	.5152	.0022	.0002	.2566	.6806	.4	-.0036	-.0050	.1444	.7608	-.0028	-.0027	.1880	.8340
TT(X)	13	-.0027	-.0041	.2500	1.031	-.0027	-.0022	.3528	14.52	8.7	-.0033	-.0043	.1581	.4808	-.0044	-.0050	.2516	4.461
TT(X,$k_n^{(x)}$)	43	-.0013	-.0024	.1596	.5082	-.0002	.0020	.1959	.4923	36	-.0011	-.0013	.1092	.7234	-.0025	-.0043	.1323	.7633
TT(X,k_n)	1	-.0011	-.0008	.2543	1.234	.0048	.0001	.6123	15.07	.4	-.0013	-.0026	.1654	.5948	-.0001	.0018	.5299	17.83
TT(Y)	1	.000	.0026	.2607	.7002	.0133	.0211	.4418	2.847	.4	-.0045	-.0003	.1656	.6793	-.0118	-.0008	.3896	4.226
		$\beta=2$				$\beta=2$					$\beta=2$				$\beta=2$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0002	-.0012	.5898	12.41	.0075	.0030	.7762	18.55	0	-.0009	-.0025	.3393	8.148	-.0038	-.0039	.9481	24.55
TT(Z)	1	.0006	-.0030	.2385	1.765	.0035	.0016	.2191	2.995	.4	-.0038	-.0051	.1747	1.354	-.0001	.0011	.1755	4.004
TT-BC(Z)	1	.0002	-.0036	.2481	1.352	.0021	.0005	.2552	1.849	.4	-.0052	-.0050	.1907	.7279	-.0006	.0011	.2078	1.764
TT(X)	13	-.0039	-.0023	.4086	5.725	.0083	.0049	.7773	18.50	8.7	-.0025	-.0026	.2754	4.063	-.0043	-.0044	.9480	24.56
TT(X,$k_n^{(x)}$)	43	.0009	.0009	.1871	1.132	.0030	.0029	.2705	3.335	36	-.0020	-.0014	.1304	.9685	-.0039	-.0011	.2128	3.518
TT(X,k_n)	1	.0017	-.0009	.4611	7.794	.0063	.0008	.7721	18.59	.4	-.0050	-.0031	.3117	6.578	.0059	-.0009	.8406	23.15
TT(Y)	1	.0034	.0051	.4635	3.195	.0186	.0236	.5712	4.562	.4	-.0033	.0006	.4149	3.886	-.0198	-.0135	.5921	5.772

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.1. (c) Estimator Properties (Symmetric Z, Known $p(X)$, Normal *or* Laplace, $n = 500$, 1000)

	$n = 500$									$n = 1000$							
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**		
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)		
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0017	.0003	.0911	.8262	-.0009	-.0007	.0966	.3476	0	-.0012	-.0014	.0065	.5459	.0003	.0008	.0679	.5371
TT(Z)	.4	.0014	.0004	.0913	.7974	-.0008	-.0013	.0945	.4294	.2	-.0010	-.0018	.0651	.5101	-.0002	-.0003	.0664	.4750
TT-BC(Z)	.4	.0014	.0005	.0913	.9478	-.0008	-.0013	.0942	.6991	.2	-.0010	-.0019	.0651	.5972	-.0002	-.0004	.0663	.4711
TT(X)	6.8	.0015	.0005	.0903	.6088	-.0002	-.0007	.0924	.5055	5.3	-.0011	-.0016	.0647	.6014	.0001	-.0005	.0653	.4377
TT(X,$k_n^{(x)}$)	32	.0006	.0020	.0741	.8656	-.0007	-.0011	.0858	.9494	29	-.0009	-.0011	.0542	.3195	-.0005	-.0002	.0613	.5140
TT(X,k_n)	.4	.0016	.0004	.0909	.6491	.0009	.0002	.0965	.6707	.2	.0005	.0003	.0643	.6475	-.0007	-.0003	.0680	.5957
TT(Y)	.4	.0035	.0046	.0911	.4930	-.0048	-.0034	.0993	.6998	.2	.0007	.0002	.0624	.6472	-.0021	-.0015	.0666	.8484
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)		
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0022	.0001	.2008	8.869	-.0004	-.0007	.2351	8.211	0	.0010	.0003	.1526	9.500	-.0025	.0003	.1721	8.006
TT(Z)	.4	-.0002	-.0002	.1023	.6013	-.0009	-.0010	.1133	.8910	.2	.0005	.0005	.0762	.6321	.0003	.0005	.0859	.6690
TT-BC(Z)	.4	-.0001	-.0002	.1026	.4637	-.0006	-.0008	.1169	.4041	.2	.0005	.0006	.0756	.7951	.0003	.0006	.0850	.6375
TT(X)	6.8	-.0013	-.0004	.1445	.2162	-.0003	-.0020	.1363	.8243	5.3	-.0002	.0004	.1035	1.956	.0001	.0017	.0975	.8950
TT(X,$k_n^{(x)}$)	32	-.0002	-.0002	.0815	.5515	-.0005	-.0007	.0981	.5972	29	.0007	.0011	.0591	.3450	-.0001	.0005	.0714	.5451
TT(X,k_n)	.4	.0002	-.0005	.1851	7.001	.0008	.0009	.2079	6.633	.2	-.0001	-.0006	.1158	5.880	-.0001	.0003	.1787	8.566
TT(Y)	.4	.0011	.0011	.1766	2.221	.0015	-.0054	.1851	1.065	.2	-.0058	-.0033	.1094	1.055	-.0097	-.0040	.1751	3.191
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)		
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0019	-.0012	.6708	23.16	.0120	.0002	.7286	21.62	0	-.0049	.0002	.8331	27.80	-.0029	.0018	.5647	21.26
TT(Z)	.4	.0006	.0006	.1017	1.078	.0008	.0011	.1240	1.303	.2	.0001	-.0002	.0835	1.202	-.0001	.0003	.1012	1.070
TT-BC(Z)	.4	.0001	.0004	.1801	.7676	-.0007	.0011	.2175	.7533	.2	-.0007	-.0002	.2397	1.143	-.0017	.0002	.1299	.7686
TT(X)	6.8	.0018	-.0010	.6707	23.15	.0023	.0016	.2958	7.045	5.3	-.0049	-.0003	.8334	27.81	.0002	.0018	.2257	6.089
TT(X,$k_n^{(x)}$)	32	.0008	.0020	.1209	1.181	-.0002	-.0009	.1301	.6615	29	-.0004	-.0004	.0953	1.032	.0002	.0009	.0982	.6948
TT(X,k_n)	.4	-.0026	-.0016	.5863	23.34	.0160	.0007	.5539	18.49	.2	.0084	-.0017	.7979	27.39	-.0006	.0042	.6425	23.50
TT(Y)	.4	-.0291	-.0002	.7267	8.0355	-.0138	-.0099	.4940	5.230	.2	.0000	-.0011	.3691	5.418	-.0076	.0030	.4959	5.935

Thetreatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.1. (d) Estimator Properties (Symmetric Z, Known $p(X)$, Normal *and* Laplace, $n = 500$, 1000)

	$n = 500$										$n = 1000$								
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$				$\beta = .25$					$\beta = .25$				$\beta = .25$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0016	.0021	.0910	.5736	.0009	.0018	.0972	.5219	0	-.0001	-.0004	.0646	.5875	-.0001	.0004	.0695	.4426
TT(Z)	.4	.0018	.0020	.0911	.4705	.0011	.0027	.0937	.7847	.2	-.0003	-.0004	.0646	.6547	.0000	.0003	.0665	.4906
TT-BC(Z)	.4	.0018	.0019	.0910	.4272	.0011	.0027	.0934	.8206	.2	-.0002	-.0004	,0646	.7702	.0000	.0003	.0664	.5873
TT(X)	6.8	.0018	.0022	.0902	.4760	.0009	.0012	.0920	.4723	5.3	-.0001	-.0004	.0642	.5853	-.0001	.0002	.0648	.4213
TT(X,$k_n^{(x)}$)	32	.0017	.0023	.0741	.8384	.0001	.0004	.0845	.4562	29	-.0005	-.0005	.0542	.5570	-.0005	-.0005	.0606	.3979
TT(X,k_n)	.4	.0017	.0031	.0900	.5943	-.0006	-.0007	.0970	.6281	.2	.0010	.0016	.0652	.5660	-.0003	.0004	.0689	.6689
TT(Y)	.4	.0031	.0021	.0919	.6245	.0005	-.0023	.0967	.5502	.2	.0012	.0016	.0653	.3621	.0004	-.0010	.0686	.6796
		$\beta = 1$				$\beta = 1$					$\beta = 1$				$\beta = 1$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-/0003	.0011	.1172	.7854	.0030	-.0002	.5274	21.33	0	.0005	.0009	.0834	.4491	-.0053	-.0016	.7248	28.73
TT(Z)	.4	-.0003	.0005	.1018	.7698	-.0007	.0005	.1080	.6676	.2	.0003	.0002	.0747	.4790	-.0007	-.0017	.0835	.6669
TT-BC(Z)	.4	-.0004	.0004	.1009	.7714	-.0008	.0006	.1067	.6909	.2	.0003	.0003	.0742	.6562	.0001	-.0017	.1121	.5042
TT(X)	6.8	-.0001	.0015	.1130	.7263	-.0021	-.0017	.1986	4.619	5.3	.0002	.0009	.0810	.3884	-.0022	-.0017	.1516	4.533
TT(X,$k_n^{(x)}$)	32	-.0005	-.0014	.0804	.7821	-.0018	-.0016	.0991	.6523	29	.0001	.0002	.0590	.6355	-.0009	-.0006	.0743	.3860
TT(X,k_n)	.4	-.0004	-.0005	.1160	.5474	-.0007	.0012	.5031	20.71	.2	.0006	.0015	.0830	.5333	-.0110	.0003	.4416	22.90
TT(Y)	.4	.0009	.0013	.1201	.7575	-.0067	.0051	.3029	3.896	.2	-.0030	-.0012	.0815	.5028	-.0059	.0007	.2035	2.599
		$\beta = 2$				$\beta = 2$					$\beta = 2$				$\beta = 2$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0052	-.0019	.2684	8.917	-.0057	.0013	.9168	26.16	0	.0000	.0015	.1643	4.783	-.0032	.0020	1.041	29.01
TT(Z)	.4	-.0022	-.0018	.1165	1.069	-.0003	.0000	.1032	1.294	.2	-.0001	.0009	.0914	.8240	.0008	.0011	.0849	1.300
TT-BC(Z)	.4	-.0018	-.0017	.1359	.6407	-.0006	.0000	.2321	.7610	.2	-.0001	.0010	.0893	.6840	.0021	.0009	.1859	.9931
TT(X)	6.8	-.0035	-.0019	.2056	3.675	-.0060	.0009	.9169	26.12	5.3	.0001	.0016	.1467	2.660	-.0033	.0018	1.041	29.01
TT(X,$k_n^{(x)}$)	32	-.0019	-.0017	.0909	.5976	.0028	.0022	.1776	3.500	29	.0005	.0017	.0739	.8953	.0020	.0027	.1487	3.389
TT(X,k_n)	.4	-.0022	-.0009	.2266	5.628	-.0102	.0010	.8791	25.92	.2	.0001	-.0008	.1607	3.984	-.0037	.0016	1.040	29.03
TT(Y)	.4	.0020	-.0023	.2564	2.713	-.0072	.0045	.5482	5.652	.2	.0005	.0003	.1640	1.689	-.0048	-.0006	.5238	4.692

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.2. (a) Rejection Frequencies (Symmetric Z, known $p(X)$, n= 100, 250)

$n = 100$							
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .052, .102	.013, .052, .099	.010, .053, .103	.011, .052, .101	.011, .051, .103	.012, .051, .104	.013,.048,.109
1	.017, .039, .068	.013, .053, .098	.011, .053, .104	.019, .055, .094	.011, .049, .100	.012, .045, .076	.019,.037,.083
2	.020, .031, .043	.018, .051, .087	.018, .052, .093	.021, .032, .044	.016, .052, .095	.021, .032, .044	.004,.004,.005
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .049, .096	.010, .052, .101	.008, .052, .104	.010, .051, .099	.011, .050, .100	.011, .048 .099	.009,.046,.103
1	.016, .034, .052	.017, .049, .090	.014, .053, .097	.016, .054, .098	.012, .051, .102	.018, .038, .058	.022,.045,.063
2	.022, .034, .045	.017, .048, .084	.017, .049, .089	.026, .046, .066	.015, .054, .098	.022, .037, .051	.025,.034,.042
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .051, .100	.010, .050, .103	.008, .051, .106	.012, .051, .100	.011, .051, .097	.011, .052, .100	.006,.046,.101
1	.013, .050, .098	.013, .049, .099	.010, .050, .104	.013, .051, .101	.011, .051, .101	.014, .054, .097	.009,.047,.089
2	.013, .026, .041	.015, .050, .092	.014, .053, .099	.025, .054, .083	.012, .052, .099	.024, .045, .069	.021,.040,.061
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .048, .093	.009, .048, .099	.008, .050, .104	.008, .049, .097	.012, .055, .098	.011, .051, .102	.011,.040,.088
1	.018, .028, .039	.014, .049, .088	.013, .052, .100	.023, .050, .081	.015, .050, .101	.017, .027, .038	.018,.039,.053
2	.020, .030, .040	.017, .047, .082	.018, .052, .093	.020, .030, .040	.018, .051, .088	.020, .030, .041	.021,.035,.043

$n = 250$							
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.001, .053, .100	.011, .052, .104	.001, .053, .107	.011, .051, .101	.010, .050, .103	.010, .053, .100	.006,.051,.100
1	.016, .036, .062	.014, .049, .096	.011, .052, .101	.018, .055, .092	.011, .048, .097	.018, .043, .075	.005,.007,.009
2	.007, .011, .014	.015, .038, .069	.018, .054, .092	.008, .011, .014	.016, .054, .095	.013, .020, .024	.012,.023,.029
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .050, .100	.009, .050, .104	.007, .051, .104	.010, .049, .104	.009, .052, .099	.001, .050, .100	.010,.055,.101
1	.013, .027, .042	.016, .050, .094	.013, .051, .099	.016, .053, .097	.011, .050, .100	.015, .034, .054	.018,.030,.044
2	.015, .022, .029	.017, .046, .081	.016, .053, .094	.025, .050, .070	.012, .054, .106	.017, .027, .036	.023,.034,.043
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .052, .099	.012, .052, .100	.010, .054, .102	.011, .051, .098	.012, .048, .097	.011, .051, .104	.011,.032,.098
1	.012, .048, .098	.010, .052, .103	.007, .051, .107	.011, .049, .099	.012, .052, .100	.010, .051, .095	.015,.054,.095
2	.017, .034, .054	.016, .049, .095	.014, .051, .100	.023, .052, .089	.012, .049, .102	.020, .040, .065	.017,.033,.042
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.01, .048, .097	.009, .050, .101	.007, .052, .105	.009, .051, .098	.011, .050, .101	.010, .049, .099	.010,.053,.103
1	.013, .021, .028	.013, .044, .084	.014, .054, .101	.022, .051, .084	.010, .051, .097	.014, .021, .029	.019,.030,.043
2	.014, .019, .023	.016, .041, .072	.018, .054, .093	.014, .019, .023	.021, .053, .090	.015, .026, .029	.022,.029,.039

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$.

Table H.2. (b) Rejection Frequencies (Symmetric Z, Known $p(X)$, n= 500, 1000)

$n = 500$

$(Y_0, Y_1, X, U) \sim$ **Normal**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.001, .050, .099	.011, .050,, .100	.001, .050, .102	.001, .050, .100	.010, .051, .100	.010, .051, .100	.012,.049,.105
1	.010, .020, .039	.013, .049, .100	.010, .050, .103	.016, .054, .096	.012, .052, .102	.010, .026, .049	.017,.029,.062
2	.013, .019, .024	.015, .051, .096	.014, .053, .100	.013, .019, .024	.013, .051, .098	.015, .022, .030	.004,.009,.016

$(Y_0, Y_1, X, U) \sim$ **Laplace**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .051, .100	.010, .047, .101	.007, .048, .104	.009, .050, .098	.009, .049, .096	.010, .050, .102	.008,.046,.093
1	.013, .027, .046	.014, .050, .097	.012, .050, .102	.013, .053, .099	.012, .049, .100	.017, .037, .063	.023,.047,.085
2	.011, .017, .022	.014, .050, .095	.014, .054, .100	.025, .051, .077	.012, .052, .099	.016, .026, .035	.010,.017,.024

$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .051, .101	.012, .051, .101	.010, .053, .102	.010, .052, .102	.010, .052, .105	.011, .050, .099	.007,.046,.103
1	.011, .049, .101	.010, .051, .101	.008, .052, .104	.011, .050, .100	.011, .050, .097	.011, .049, .100	.012,.065,.097
2	.013, .030, .048	.014, .051, .101	.011, .053, .103	.023, .051, .086	.011, .052, .098	.020, .045, .072	.014,.027,.046

$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .051, .097	.010, .051, .096	.008, .052, .100	.011, .051, .098	.011, .049, .097	.012, .052, .099	.014,.049,.100
1	.010, .016, .021	.014, .048, .095	.011, .050, .099	.024, .052, .083	.011, .049, .099	.011, .017, .023	.018,.030,.042
2	.010, .015, .021	.013, .049, .093	.013, .052, .101	.010, .015, .020	.021, .054, .089	.011, .016, .021	.019,.029,.040

$n = 1000$

$(Y_0, Y_1, X, U) \sim$ **Normal**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .050, .101	.011, .049, .102	.009, .050, .104	.101, .049, .099	.011, .050, .099	.010, .050, .102	.014,.050,.089
1	.009, .018, .031	.011, .049, .099	.008, .051, .100	.015, .052, .093	.010, .048, .102	.016, .040, .076	.017,.048,.086
2	.007, .011, .015	.016, .052, .098	.015, .055, .100	.007, .011, .014	.013, .052, .099	.008, .012, .015	.018,.026,.034

$(Y_0, Y_1, X, U) \sim$ **Laplace**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .051, .099	.011, .052, .099	.009, .052, .103	.010, .052, .101	.009, .051, .101	.011, .050, .100	.006,.043,.095
1	.014, .027, .046	.010, .047, .100	.008, .050, .105	.013, .048, .095	.011, .048, .099	.012, .026, .040	.017,.026,.037
2	.012, .019, .025	.015, .053, .097	.012, .054, .100	.025, .050, .075	.011, .052, .100	.010, .015, .019	.013,.020,.026

$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.010, .049, .100	.010, .047, .100	.008, .048, .103	.010, .049, .100	.009, .051, .100	.010, .050, .102	.008,.051,.111
1	.010, .049, .103	.010, .050, .100	.007, .051, .102	.010, .049, .102	.010, .049, .100	.009, .050, .101	.013,.053,.102
2	.017, .040, .072	.012, .051, .098	.009, .052, .104	.018, .052, .091	.009, .052, .100	.020, .047, .078	.019,.051,.078

$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**

β	No Trim	TT(Z)	TT–BC(Z)	TT(X)	TT(X,$k_n^{(x)}$)	TT(X,k_n)	TT(Y)
.25	.011, .048, .097	.009, .048, .104	.006, .048, .105	.009, .051, .100	.010, .049, .100	.012, .049, .097	.011,.053,.105
1	.005, .007, .009	.011, .050, .100	.009, .051, .104	.024, .052, .083	.011, .052, .100	.011, .015, .019	.026,.045,.066
2	.009, .012, .016	.013, .052, .096	.012, .053, .102	.009, .013, .016	.022, .056, .093	.009, .012, .016	.018,.027,.038

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. "TT(Y)" is $\hat{\theta}_n^{(ty)}$.

Table H.3. (a) Trim-by-$p(X)$ Estimator Properties (Symmetric Z, Known $p(X)$, Normal *or* Laplace, $n = 100$, 250)

	$n=100$										$n=250$								
		$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**					$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0023	.0025	.2027	.6031	.0018	.0020	.2179	.5773	0	-.0004	-.0006	.1289	.4570	.0015	.0018	.1366	.4855
.25ln(n)	2	.0023	.0017	.1999	.6515	.0013	.0030	.2143	.7167	.8	.0009	.0019	.1277	.5522	.0015	.0020	.1356	.5091
.5n/ln(n)	10	-.0003	.0021	.1942	.7924	.0001	-.0014	.2076	.5621	8.8	.0015	.0013	.1226	.7978	-.0011	-.0001	.1303	.5623
n/ln(n)	22	-.0020	-.0027	.1777	.4305	.0018	-.0015	.1996	.7519	18.4	-.0007	-.0010	.1165	.6106	-.0003	-.0007	.1274	.2959
2n/ln(n)	44	-.0001	-.0009	.1506	.5428	.0001	-.0002	.1832	.5591	36	.0004	.0008	.1014	.5305	-.0004	-.0002	.1198	.4448
4n/ln(n)	86	-.0008	-.0007	.0747	.4618	.0016	.0018	.1016	.7838	72.8	.0000	-.0005	.0667	.6269	-.0012	-.0015	.0870	.5036
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0071	.0047	.3376	5.751	.0013	.0017	.4556	8.912	0	.0001	-.0021	.2302	5.632	-.0041	-.0036	.3351	10.21
.25ln(n)	2	-.0001	-.0013	.3237	4.339	.0001	-.0006	.3837	5.613	.8	.0008	.0010	.2172	4.187	-.0013	-.0029	.2751	6.431
.5n/ln(n)	10	-.0019	-.0022	.2433	1.221	-.0018	-.0028	.2921	2.355	8.8	-.0012	-.0009	.1498	.6107	.0007	.0001	.1895	1.205
n/ln(n)	22	.0007	.0017	.2041	.6709	-.0024	-.0004	.2457	1.070	18.4	-.0004	.0005	.1358	.6605	.0011	.0013	.1605	.6505
2n/ln(n)	44	.0009	-.0009	.1602	.5833	-.0025	-.0016	.1953	.5840	36	.0008	.0005	.1108	.9013	-.0003	-.0011	.1319	.6192
4n/ln(n)	86	.0007	.0016	.0757	.9814	-.0022	-.0029	.1024	.7040	72.8	.0006	.0006	.0666	.6158	.0011	.0023	.0882	.7080
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0001	-.0010	.6623	16.54	-.0014	.0053	.7859	16.06	0	.0097	.0009	1.137	27.47	-.0021	-.0028	.7826	19.56
.25ln(n)	2	-.0001	.0013	1.614	28.75	-.0156	.0031	3.621	34.12	.8	.0500	.0018	4.304	38.14	.0148	-.0015	1.047	23.78
.5n/ln(n)	10	.0015	-.0083	1.109	22.91	.0037	.0059	.6045	11.83	8.8	-.0190	-.0060	.8258	23.61	.0023	.0045	.4160	9.336
n/ln(n)	22	.0010	-.0001	.4165	9.391	-.0001	.0020	.3790	4.569	18.4	.0004	.0017	.3050	8.140	.0030	.0068	.2664	3.712
2n/ln(n)	44	-.0019	-.0010	.2088	2.093	-.0022	-.0036	.2333	1.315	36	.0023	.0035	.1651	1.654	.0005	.0019	.1706	.8549
4n/ln(n)	86	.0008	.0010	.0773	.3035	-.0007	-.0012	.1037	1.103	72.8	.0006	.0003	.0689	.6176	-.0009	-.0001	.0919	.5810

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25\ln(n)$,...,$4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.3. (b) Trim-by-$p(X)$ Estimator Properties (Symmetric Z, Known $p(X)$, Normal *and* Laplace, $n = 100, 250$)

$n = 100$

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$				$\beta = .25$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0005	.0003	.2054	.7790	.0001	.0031	.2189	.7417
.25ln(n)	2	.0005	-.0015	.2023	.5211	-.0022	-.0019	.2131	.7668
.5n/ln(n)	10	.0015	.0021	.1931	.3600	.0011	.0016	.2036	.5834
n/ln(n)	22	-.0016	-.0017	.1800	.5264	-.0010	.0017	.1975	.6824
2n/ln(n)	44	.0001	-.0003	.1489	.6108	.0027	.0025	.1807	.4144
4n/ln(n)	86	-.0001	-.0006	.0745	.5699	.0023	.0026	.1018	.7991
		$\beta = 1$				$\beta = 1$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0019	-.0031	.2637	1.190	-.0041	-.0050	.5865	14.20
.25ln(n)	2	-.0025	-.0072	.2513	.8399	.0044	-.0010	1.019	22.90
.5n/ln(n)	10	-.0021	.0015	.2289	.9670	-.0041	-.0009	.3838	6.282
n/ln(n)	22	.0016	.0016	.2020	.4042	-.0013	-.0001	.2677	1.747
2n/ln(n)	44	-.0009	-.0015	.1578	.4519	.0025	.0021	.1960	.5463
4n/ln(n)	86	.0013	.0020	.0752	.7878	.0003	.0005	.1030	.7984
		$\beta = 2$				$\beta = 2$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0002	-.0012	.5898	12.41	.0075	.0030	.7762	18.55
.25ln(n)	2	.0040	-.0005	.4560	7.864	.0031	.0009	1.432	31.32
.5n/ln(n)	10	.0001	-.0008	.3402	3.328	-.0020	.0008	.6710	15.67
n/ln(n)	22	-.0036	-.0017	.2686	1.729	-.0003	-.0006	.4011	4.299
2n/ln(n)	44	.0011	-.0004	.1851	.9053	.0013	-.0002	.2444	1.599
4n/ln(n)	86	-.0012	-.0007	.0075	.7386	.0010	.0012	.1220	.8880

$n = 250$

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$				$\beta = .25$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0002	-.0009	.1296	.5263	.0015	.0017	.1388	.5094
.25ln(n)	.8	.0007	.0001	.1267	.4010	-.0005	.001	.1362	.6012
.5n/ln(n)	8.8	-.0003	.0008	.1245	.7627	-.0011	-.0027	.1316	.6763
n/ln(n)	18.4	-.0005	-.0017	.1153	.5523	.0017	.0008	.1257	.5597
2n/ln(n)	36	-.0002	-.0009	.1017	.5329	.0002	.0010	.1192	.7281
4n/ln(n)	72.8	-.0006	-.0003	.0660	.5294	.0003	.0006	.0871	.4541
		$\beta = 1$				$\beta = 1$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0035	-.0068	.1164	.8191	-.0108	-.0045	.5499	18.24
.25ln(n)	.8	-.0017	-.0027	.1655	.5265	-.1178	-.0017	11.71	42.35
.5n/ln(n)	8.8	-.0025	-.0015	.1435	.5055	.045	.0051	.2928	7.873
n/ln(n)	18.4	-.0010	-.0009	.1313	.6464	.0018	.0011	.1795	1.252
2n/ln(n)	36	.0005	.0010	.1106	.4502	-.0003	-.0001	.1336	.3679
4n/ln(n)	72.8	.0005	.0007	.0669	.4266	-.0007	-.0011	.0865	.7756
		$\beta = 2$				$\beta = 2$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0009	-.0025	.3393	8.148	-.0038	-.0039	.9481	24.55
.25ln(n)	.8	-.0026	-.0022	.3132	8.838	-.1827	-.0024	14.56	45.66
.5n/ln(n)	8.8	.0046	.0033	.2185	1.647	-.0607	-.0302	2.572	16.43
n/ln(n)	18.4	.0003	-.0018	.1778	1.062	.0045	.0008	.9768	4.321
2n/ln(n)	36	.0023	.0034	.1321	.6439	-.0007	.0003	.1747	1.023
4n/ln(n)	72.8	-.0011	-.0006	.0701	.4487	.0001	.0002	.0945	.7864

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. “No Trim” is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25\ln(n)$,...,$4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.3. (c) Trim-by-$p(X)$ Estimator Properties (Symmetric Z, Known $p(X)$, Normal *or* Laplace, $n = 500$, 1000)

	$n = 500$									$n = 1000$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0017	.0003	.0911	.8262	-.0009	-.0007	.0966	.3476	0	-.0012	-.0014	.0065	.5459	.0003	.0008	.0679	.5371
.25ln(n)	.4	-.0008	.0001	.0906	.6841	.0001	.0008	.0975	.7388	.2	.0009	.0006	.0654	.3802	-.0002	-.0010	.0688	.6138
.5n/ln(n)	8	-.0003	-.0012	.0875	.4091	-.0004	-.0017	.0927	.5754	7.2	-.0002	-.0004	.0619	.4683	-.0006	-.0008	.0661	.5372
n/ln(n)	16	-.0016	-.0020	.0832	.4686	.0009	.0014	.0913	.7179	14.4	-.0009	-.012	.0600	.6389	-.0001	.0001	.0646	.4974
2n/ln(n)	32	.0005	.0006	.0749	.4629	.0007	.0009	.0862	.5395	29	-.0008	-.0006	.0534	.3872	-.0006	-.0001	.0606	.8303
4n/ln(n)	64.4	-.0003	-.0008	.0546	.5814	-.0010	-.0008	.0686	.7069	58	-.0009	-.0016	.0410	.6761	-.0006	-.0007	.0513	.6192
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0022	.0001	.2008	8.869	-.0004	-.0007	.2351	8.211	0	.0010	.0003	.1526	9.500	-.0025	.0003	.1721	8.006
.25ln(n)	.4	.0020	.0013	.1607	3.942	-.0030	-.0014	.2248	7.617	.2	.0011	.0007	.1240	5.215	-.0014	-.0009	.1540	.5470
.5n/ln(n)	8	.0006	.0004	.1124	.6341	-.0030	-.0042	.1370	.8712	7.2	.0001	-.0002	.0812	.4924	.0001	.0016	.0986	.8403
n/ln(n)	16	.0009	.0012	.0989	.7365	.0005	.0005	.1181	.5005	14.4	.0004	-.0009	.0726	.7493	.0017	.0016	.0851	.6505
2n/ln(n)	32	.0004	.0002	.0822	.3613	.0003	.0001	.0986	.7262	29	-.0008	-.0016	.0606	.7938	-.001	.0001	.0724	.4345
4n/ln(n)	64.4	.0008	.0008	.0542	.6310	-.0007	-.0006	.0700	.4581	58	-.0001	.0003	.0424	.4757	-.0001	-.0004	.0539	.4337
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0019	-.0012	.6708	23.16	.0120	.0002	.7286	21.62	0	-.0049	.0002	.8331	27.80	-.0029	.0018	.5647	21.26
.25ln(n)	.4	.0049	-.0009	1.035	28.03	.0039	-.0001	.7506	22.16	.2	-.0070	-.0007	.9978	29.51	.0133	.0023	.6356	22.84
.5n/ln(n)	8	.0082	-.0005	.5248	19.68	.0019	.0027	.2965	7.163	7.2	-.0047	.0009	.3885	17.73	.0015	.0018	.2261	6.795
n/ln(n)	16	-.0001	.0020	.2641	8.893	.0010	.0001	.2003	2.651	14.4	.0027	.0007	.2041	7.473	-.0011	-.0007	.1512	.2145
2n/ln(n)	32	-.0004	-.0003	.1226	1.264	-.0023	-.0031	.1308	.6148	29	-.0002	.0006	.0949	.8150	-.0011	-.0023	.0990	.5703
4n/ln(n)	64.4	.0001	.0001	.0590	.5348	-.0016	-.0013	.0746	.5338	58	.0001	.0003	.0474	.5915	-.0003	.0001	.0581	.4972

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25\ln(n)$,...,$4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.3. (d) Trim-by-$p(X)$ Estimator Properties (Symmetric Z, Known $p(X)$, Normal *and* Laplace, $n = 500$, 1000)

	$n = 500$									$n = 1000$								
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$				$\beta = .25$					$\beta = .25$				$\beta = .25$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0016	.0021	.0910	.5736	.0009	.0018	.0972	.5219	0	-.0001	-.0004	.0646	.5875	-.0001	.0004	.0695	.4426
.25ln(n)	.4	.0009	.0011	.0913	.7226	.0002	.0014	.0963	.6180	.2	.0004	.0003	.0653	.5949	-.0014	-.0015	.0700	.6258
.5n/ln(n)	8	-.0003	.0002	.0870	.7392	-.0011	-.0016	.0921	.4101	7.2	.0006	.0010	.0618	.6586	-.0008	-.0009	.0651	.5257
n/ln(n)	16	.0001	.0009	.0824	.5050	-.003	-.0007	.0911	.4293	14.4	-.0003	-.0001	.0590	.6013	.0003	.0006	.0635	.5113
2n/ln(n)	32	-.0003	.0001	.0742	.5673	-.0001	-.0009	.0855	.7464	29	-.0002	-.0001	.0544	.5617	.0008	.0007	.0609	.7678
4n/ln(n)	64.4	.0001	-.0001	.0533	.5281	-.0004	-.0005	.0681	.3891	58	.0003	.0002	.0413	.7114	-.0006	.0002	.0515	.8883
		$\beta = 1$				$\beta = 1$					$\beta = 1$				$\beta = 1$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0003	.0011	.1172	.7854	.0030	-.0002	.5274	21.33	0	.0005	.0009	.0834	.4491	-.0053	-.0016	.7248	28.73
.25ln(n)	.4	.0007	.0016	.1168	.5587	-.0180	.0010	1.533	33.19	.2	-.0004	-.0001	.0814	.5288	.0097	.0022	.5317	22.73
.5n/ln(n)	8	-.0005	.0001	.1040	.5798	-.0002	-0020	.2103	5.594	7.2	.0002	.0002	.0748	.4185	-.0015	-.0040	.1521	4.408
n/ln(n)	16	.0019	.0022	.0956	.3647	-.0023	.0001	.1345	1.423	14.4	.0006	.0010	.0678	.6343	-.0010	-.0010	.1004	.8132
2n/ln(n)	32	-.0005	.0001	.0812	.5054	.0001	-.00013	.1000	.5341	29	-.0001	-.0014	.0595	.9357	.0002	-.0004	.0728	.4556
4n/ln(n)	64.4	.0002	.0002	.0545	.4886	.0003	.0006	.0700	.4799	58	.0001	-.0001	.0434	.6837	-.0005	-.0003	.0531	.7506
		$\beta = 2$				$\beta = 2$					$\beta = 2$				$\beta = 2$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0052	-.0019	.2684	8.917	-.0057	.0013	.9168	26.16	0	.0000	.0015	.1643	4.783	-.0032	.0020	1.041	29.01
.25ln(n)	.4	-.0019	-.0016	.2222	4.825	.0204	.0050	2.402	44.82	.2	.0014	.0004	.1623	4.511	.0078	.0032	1.756	31.03
.5n/ln(n)	8	.0004	.0012	.1593	1.063	.0057	.0033	1.029	14.54	7.2	.0005	-.0008	.1121	.7481	.0022	.0017	1.047	8.134
n/ln(n)	16	.0006	-.0014	.1323	.7286	-.0022	-.0015	.7874	4.322	14.4	.0005	.0003	.0955	.5667	-.0011	-.0007	.5642	2.542
2n/ln(n)	32	-.0004	.0006	.0996	.7129	-.0006	.0001	.1786	1.203	29	-.0006	-.0010	.0728	.5019	-.0008	-.0007	.1143	.8677
4n/ln(n)	64.4	.0007	.0015	.0581	.7457	.0003	.0003	.0837	.9973	58	-.0003	.0001	.0459	.3968	.0006	.0004	.0675	.9826

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25\ln(n)$,...,$4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.4. (a) Trim-by-$p(X)$ Rejection Frequencies (Symmetric Z, Known $p(X)$, n= 100, 250)

$n = 100$						
$(Y_0, Y_1, X, U) \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .052, .102	.011, .052, .102	.001, .049, .098	.011, .050, .100	.011, .049, .100	.011, .049, .098
1	.017, .039, .068	.017, .051, .073	.015, .052, .097	.010, .051, .102	.011, .048, .098	.010, .049, .097
2	.020, .031, .043	.008, .012, .014	.001, .018, .022	.025, .042, .061	.017, .049, .087	.011, .048, .099
$(Y_0, Y_1, X, U) \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .049, .096	.012, .051, .100	.010, .049, .103	.012, .049, .097	.011, .049, .098	.013, .050, .100
1	.016, .034, .052	.018, .042, .067	.018, .052, .095	.015, .052, .100	.011, .048, .098	.013, .051, .098
2	.022, .034, .045	.002, .004, .004	.022, .038, .054	.022, .053, .077	.014, .053, .098	.013, .054, .101
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .051, .100	.011, .048, .099	.001, .049, .102	.009, .052, .101	.011, .053, .101	.011, .049, .099
1	.013, .050, .098	.012, .052, .096	.012, .050, .102	.010, .050, .104	.010, .050, .099	.011, .051, .102
2	.013, .026, .041	.011, .023, .033	.020, .032, .049	.015, .041, .089	.012, .051, .104	.011, .049, .099
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .048, .093	.012, .050, .103	.011, .052, .103	.011, .052, .103	.010, .051, .100	.013, .051, .097
1	.018, .028, .039	.008, .012, .016	.020, .041, .064	.017, .051, .093	.011, .049, .097	.012, .051, .098
2	.020, .030, .040	.005 .008, .009	.009, .014, .019	.024, .043, .060	.016, .052, .093	.010, .052, .102

$n = 250$						
$(Y_0, Y_1, X, U) \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.001, .053, .100	.010, .048, .098	.010, . 049, .102	.009, .049, .098	.010, .049, .096	.009, .049, .098
1	.016, .036, .062	.016, .039, .069	.012, .047, .095	.010, .050, .099	.011, .053, .101	.010, .047, .100
2	.007, .011, .014	.002, .003, .003	.013, .019, .025	.026, .048, .069	.017, .053, .094	.011, .050, .103
$(Y_0, Y_1, X, U) \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .050, .100	.011, .051, .100	.011, .051, .103	.010, .051, .101	.011, .051, .099	.011, .051, .100
1	.013, .027, .042	.017, .039, .064	.016, .052, .097	.013, .052, .102	.011, .053, .102	.011, .049, .098
2	.015, .022, .029	.009, .013, .017	.027, .047, .068	.022, .054, .088	.013, .049, .096	.011, .049, .099
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .052, .099	.010, .051, .103	.012, .049, .098	.010, .052, .098	.010, .048, .099	.011, .048, .097
1	.012, .048, .098	.011, .051, .099	.011, .054, .099	.009, .049, .099	.010, .052, .102	.010, .047, .102
2	.017, .034, .054	.006, .012, .019	.011, .020, .031	.027, .039, .063	.013, .047, .094	.011, .050, .102
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.01, .048, .097	.011, .049, .098	.010, .052, .100	.011, .052, .101	.011, .049, .099	.010, .048, .100
1	.013, .021, .028	.013, .019, .032	.018, .036, .055	.014, .049, .097	.011, .049, .098	.011, .051, .102
2	.014, .019, .023	.011, .015, .020	.013, .018, .023	.026, .044, .065	.015, .052, .098	.010, .051, .099

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25ln(n)$,...,$4n/ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

Table H.4. (b) Trim-by-$p(X)$ Rejection Frequencies (Symmetric Z, Known $p(X)$, n= 500, 1000)

$n = 500$						
$(Y_0, Y_1, X, U) \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.001, .050, .099	.011, .050, .101	.010, .050, .101	.011, .050, .101	.011, .049, .099	.010, .050, .101
1	.010, .020, .039	.018, .039, .069	.011, .050, .096	.010, .051, .101	.010, .049, .098	.010, .051, .099
2	.013, .019, .024	.008, .011, .015	.017, .026, .033	.026, .048, .071	.012, .054, .102	.011, .051, .101
$(Y_0, Y_1, X, U) \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .051, .100	.010, .051, .102	.011, .051, .101	.012, .050, .100	.011, .049, .097	.009, .048, .100
1	.013, .027, .046	.016, .032, .052	.014, .052, .095	.011, .049, .101	.009, .048, .099	.011, .051, .098
2	.011, .017, .022	.009, .015, .021	.027, .052, .077	.019, .052, .091	.013, .049, .097	.011, .051, .098
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .051, .101	.009, .052, .101	.010, .051, .101	.010, .050, .098	.012, .049, .097	.012, .052, .100
1	.011, .049, .101	.011, .052, .097	.011, .047, .100	.010, .053, .099	.011, .051, .096	.010 .051, .101
2	.013, .030, .048	.023, .045, .065	.021, .056, .087	.015, .054, .101	.012, .049, .098	.009, .050, .099
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .051, .097	.013, .047, .096	.010, .053, .098	.009, .053, .102	.009, .049, .100	.012, .049, .098
1	.010, .016, .021	.003, .004, .051	.021, .045, .070	.014, .048, .096	.011, .052, .099	.009, .049, .099
2	.010, .015, .021	.005, .009, .012	.005, .012, .023	.021, .054, .076	.016, .055, .096	.012, .051, .102

$n = 1000$						
$(Y_0, Y_1, X, U) \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .050, .101	.010, .051, .101	.010, .048, .099	.010, .045, .098	.011, .050, .099	.010, .050, .098
1	.009, .018, .031	.013, .032, .060	.011, .052, .101	.012, .049, .098	.011, .050, .100	.010, .051, .099
2	.007, .011, .015	.006, .010, .012	.021, .031, .040	.027, .051, .073	.012, .052, .100	.011, .049, .100
$(Y_0, Y_1, X, U) \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .051, .099	.011, .052, .099	.010, .051, .104	.012, .048, .096	.011, .049, .104	.010, .049, .098
1	.014, .027, .046	.017, .037, .636	.013, .051, .098	.011, .049, .096	.011, .048, .099	.010, .048, .102
2	.012, .019, .025	.009, .014, .019	.027, .053, .082	.020, .057, .094	.011, .048, .098	.010, .048, .099
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.010, .049, .100	.008, .049, .101	.012, .051, .098	.011, .051, .098	.011, .050, .098	.009, .048, .100
1	.010, .049, .103	.010, .048, .100	.011, .050, .100	.012, .050, .099	.012, .051, .098	.009, .053, .103
2	.017, .040, .072	.021, .045, .073	.012, .052, .095	.012, .051, .099	.011, .051, .101	.010, .049, .101
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**						
β	No Trim	.025ln(n)	.5n/ln(n)	n/ln(n)	2n/ln(n)	4n/ln(n)
.25	.011, .048, .097	..010, .051, .100	.010, .049, .099	.011, .048, .099	.009, .051, .099	.009, .047, .097
1	.005, .007, .009	.007, .011, .013	.022, .051, .081	.012, .052, .099	.009, .049, .099	.009, .049, .101
2	.009, .012, .016	.002, .006, .011	.004, .012, .019	.006, .024, .045	.021, .047, .081	.014, .048, .098

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. All values $\{25ln(n)$,...,$4n/ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

H.2 One Covariate, Known $p(X)$, and Asymmetric Z

Table H.5. (a) Estimator Properties (Asymmetric Z, Known $p(X)$, $n = 100, 250$)

	$n = 100$									$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0019	.0016	.2074	.5265	.0009	.0033	.2254	.6493	0	-.0013	-.0019	.1315	.3786	-.0026	-.0036	.1439	.5307
TT-BC(Z)	1	.0019	.0033	.2058	.7577	.0010	.0006	.2175	.5957	.4	-.0011	.0001	.1303	.6102	-.0024	-.0018	.1383	.5104
TT(X,$k_n^{(x)}$)	43	.0023	.0023	.1549	.6590	-.0006	-.0009	.1894	.4123	36	-.0008	-.0011	.1020	.4841	-.0015	-.0022	.1237	.6083
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0045	.0029	.3581	6.455	.0041	.0026	.4771	.9200	0	-.0012	.0008	.2481	6.980	.0014	-.0017	.4005	13.72
TT-BC(Z)	1	.0050	.0074	.2155	.6294	.0037	.0006	.2378	.6198	.4	.0005	.0019	.1468	.5941	-.0005	.0002	.1630	.6294
TT(X,$k_n^{(x)}$)	43	.0028	.0033	.1636	.4708	.0018	.0002	.1986	.9803	36	.0009	.0003	.1130	.4695	-.008	.0005	.1365	.4900
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0048	.0001	.9474	21.81	.0101	.0044	.7679	15.48	0	-.0052	-.0019	.7042	20.82	-.0058	.0029	.7880	20.01
TT-BC(Z)	1	.0012	.0006	.2582	2.182	.0008	.0024	.2727	1.793	.4	-.0002	-.0001	.2202	1.786	.0026	.0011	.2731	.9982
TT(X,$k_n^{(x)}$)	43	.0004	.0009	.2161	2.603	.0014	.0029	.2428	1.778	36	-.0005	-.0012	.1602	1.862	.0020	.0010	.1731	1.055

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0018	-.0016	.2076	.3912	.0023	.0020	.2157	.6367	0	-.0009	-.0021	.1287	.4928	-.0011	-.0011	.1365	.7204
TT-BC(Z)	1	-.0029	-.0044	.2089	.5164	.0033	.0043	.2082	.5288	.4	-.0008	-.0004	.1292	.4356	-.0013	-.0021	.1306	.9142
TT(X,$k_n^{(x)}$)	43	-.0013	-.0005	.1553	.5728	.0001	.0013	.1769	.7379	36	-.0004	-.0015	.1017	.6008	-.0006	-.0001	.1174	.6657
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0034	-.0042	.2661	1.131	.0014	-.0008	.7626	17.69	0	-.0029	-.0050	.1696	.9470	-.0089	-.0032	.7094	.2153
TT-BC(Z)	1	-.0026	-.0032	.2171	.4071	.0014	.0007	.2418	.7800	.4	-.0037	-.0036	.1465	.6245	-.0009	-.0003	.1744	.9857
TT(X,$k_n^{(x)}$)	43	-.0005	-.0009	.1610	.6371	-.0006	-.0006	.1976	.4325	36	-.0017	-.0026	.1095	.5110	-.0011	-.0030	.1328	.6897
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0012	-.0046	.5675	11.22	-.0028	.0001	.8648	19.23	0	.0007	-.0003	.3290	7.072	.0117	.0019	1.088	25.13
TT-BC(Z)	1	.0008	.0001	.2734	1.231	-.0013	.0019	.2795	1.872	.4	.0005	.0030	.1735	.7289	-.0003	.0038	.2448	2.154
TT(X,$k_n^{(x)}$)	43	.0001	.0002	.1890	1.085	.0003	-.0010	.2759	2.754	36	-.0003	-.0005	.1317	.7510	.0013	.0049	.2168	3.345

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.5. B. Estimator Properties (Asymmetric Z, Known $p(X)$, $n = 500$, 1000)

	$n = 500$									$n = 1000$									
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0019	.0003	.0928	.7692	-.0002	-.0010	.1026	.9724	0	.0007	-.0004	.0661	.6902	.0003	.0002	.0710	.7295
TT-BC(Z)	.4	.0017	.0009	.0919	.8483	-.0001	-.0006	.0982	.5354	.2	.0007	.0002	.0654	.7915	.0001	.0001	.0688	.5324
TT(X,$k_n^{(x)}$)	32	.0010	.0014	.0754	.6260	-.0005	-.0006	.0896	.4572	29	.0007	.0004	.0547	.4123	-.0010	-.0002	.0634	.5462
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0008	-.0002	.1958	7.667	-.0014	-.0010	.2895	12.20	0	.0006	-.0004	.1406	7.127	-.0007	-.0006	.2216	13.17
TT-BC(Z)	.4	.0001	.0001	.1011	.5093	.0009	.0001	.1135	.4447	.2	.0004	.0003	.0761	.5442	.0005	-.0002	.0861	.4560
TT(X,$k_n^{(x)}$)	32	.0002	.0006	.0831	.5930	-.0001	-.0005	.1007	.5446	29	.0008	.0005	.0613	.4559	.0006	.0003	.0733	.6904
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0116	.0034	.7714	25.11	-.0027	-.0017	.6663	20.18	0	-.0101	.0005	1.171	31.23	-.0078	-.0021	.0697	22.40
TT-BC(Z)	.4	-.0009	.0011	.1482	.8110	-.0013	-.0021	.1402	1.067	.2	-.0015	-.0001	.1419	.7565	-.0015	.0004	.1365	.7851
TT(X,$k_n^{(x)}$)	32	-.0007	-.0004	.1276	1.900	.0008	.0005	.1330	.8990	29	.0002	.0002	.1005	1.632	-.0002	-.0001	.0997	.6287

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0006	.0001	.0925	.4228	-.0004	-.0008	.0962	.6322	0	-.0002	-.0001	.0657	.4438	-.0003	-.0007	.0696	.6268
TT-BC(Z)	.4	-.0005	-.0004	.0925	.5925	-.0006	-.0008	.0927	.6779	.2	-.002	-.0003	.0656	.5262	.0003	-.0005	.0664	.8032
TT(X,$k_n^{(x)}$)	32	.0001	.0001	.0751	.4632	-.0007	-.0013	.0842	.5852	29	-.0002	-.0001	.0541	.4046	-.0003	-.0004	.0607	.8789
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0001	.0015	.1183	.6271	-.0073	.0014	.5306	20.97	0	.0007	.0011	.0848	.7072	.0011	-.0004	.6263	25.51
TT-BC(Z)	1	-.0001	.0005	.1028	.6269	-.0003	.0018	.1502	.8700	.2	.0007	.0011	.0753	.7229	.0004	.0009	.0909	.5970
TT(X,$k_n^{(x)}$)	43	-.0003	.0003	.0816	.4524	-.0001	.0006	.0990	.8007	29	.0006	.0006	.0599	.4948	.0003	.0012	.0725	.7388
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0053	-.0026	.2780	9.444	-.0053	.0004	1.055	26.39	0	-.0027	-.0018	.1758	5.459	-.0082	.0039	1.254	29.72
TT-BC(Z)	1	-.0019	-.0009	.1317	.5329	.0017	-.0007	.2296	1.460	.2	-.0006	-.0013	.0923	.4658	.0018	.0014	.2448	1.199
TT(X,$k_n^{(x)}$)	43	-.0023	-.0012	.1005	1.002	.0016	.0020	.1846	3.298	29	.0006	.0004	.0744	.6562	-.0001	.0014	,1574	3.322

The treatment assignment is $D = I(.25 + \beta X > U)$ with $\alpha = 0$, hence Z has a asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.6. Rejection Frequencies (Asymmetric Z, Known $p(X)$

$n = 100$					$n = 250$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .050, .102	.007, .049, .103	.010, .051, .099	.25	.010, .052, .098	.007, .050, .103	.011, .051, .099
1	.018, .037, .060	.010, .054, .103	.011, .049, .104	1	.014, .033, .056	.010, .053, .103	.011, .049, .098
2	.011, .017, .023	.016, .051, .092	.020, .053, .094	2	.014, .023, .029	.017, .050, .092	.017, .053, .095
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .049, .097	.007, .050, .104	.011, .051, .100	.25	.010, .050, .099	.007, .049, .103	.012, .051, .099
1	.016, .031, .048	.013, .053, .098	.011, .051, .097	1	.009, .018, .025	.013, .054, .102	.011, .052, .101
2	.021, .032, .044	.017, .052, .092	.016, .054, .096	2	.015, .022, .029	.017, .054, .095	.014, .049, .096
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .049, .101	.008, .050, .103	.010, .049, .101	.25	.010, .051, .099	.008, .052, .102	.011, .050, .101
1	.013, .052, .098	.009, .051, .103	.010, .052, .100	1	.014, .049, .098	.009, .052, .100	.010, .051, .100
2	.016, .029, .043	.016, .054, .099	.013, .053, .103	2	.022, .042, .065	.014, .054, .101	.011, .054, .106
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .052, .099	.008, .051, .104	.011, .051, .100	.25	.011, .053, .100	.007, .050, .101	.012, .050, .102
1	.015, .022, .029	.013, .052, .101	.010, .048, .101	1	.010, .015, .020	.014, .053, .101	.012, .051, .102
2	.020, .029, .040	.017, .049, .090	.020, .052, .089	2	.014, .021, .026	.019, .055, .091	.021, .055, .093

$n = 500$					$n = 1000$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .052, .101	.007, .051, .103	.010, .052, .101	.25	.010, .051, .100	.008, .050, .103	.010, .051, .100
1	.011, .025, .043	.008, .053, .105	.010, .050, .098	1	.013, .028, .050	.008, .050, .101	.011, .050, .102
2	.009, .014, .018	.014, .053, .099	.017, .053, .094	2	.006, .008, .010	.015, .052, .100	.015, .055, .098
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .052, .103	.010, .0483, .099	.009, .053, .100	.25	.011, .049, .097	.008, .052, .102	.009, .049, .105
1	.009, .018, .028	.011, .052, .101	.010, .050, .101	1	.007, .014, .024	.010, .052, .101	.009, .049, .102
2	.012, .020, .027	.014, .054, .102	.013, .051, .102	2	.012, .017, .022	.012, .053, .101	.012, .051, .097
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .051, .098	.009, .052, .101	.010, .051, .101	.25	.011, .051, .102	.009, .050, .103	.010, .047, .098
1	.011, .050, .099	.008, .050, .101	.010, .052, .101	1	.011, .050, .100	.008, .050, .104	.011, .049, .102
2	.013, .028, .045	.011, .051, .102	.011, .051, .099	2	.018, .041, .072	.009, .053, .104	.010, .053, .101
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .049, .098	.007, .048, .103	.010, .049, .098	.25	.010, .049, .098	.008, .052, .101	.010, .049, .097
1	.014, .021, .027	.011, .052, .102	.011, .051, .100	1	.009, .013, .015	.012, .052, .102	.010, .049, .099
2	.011, .017, .021	.015, .051, .096	.021, .057, .089	2	.010, .013, .017	.014, .053, .100	.022, .055, .091

The treatment assignment is $D = I(.25+\beta X > U)$, hence Z has an asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$.

Table H.7. (a) Trim-by-$p(X)$ Estimator Properties (Asymmetric Z, Known $p(X)$, $n = 100$, 250)

	$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0019	.0014	.2033	.4219	-.0015	-.0015	.2230	1.119	0	-.0017	-.0032	.1294	.7484	-.0012	-.0012	.1434	.4545
2n/ln(n)	44	.0005	.0009	.1506	.5689	-.0023	-.0002	.1868	.6596	36	-.0010	-.0012	.1015	.5845	-.0011	-.0022	.1235	.9964
4n/ln(n)	86	-.0008	.0003	.0751	.7752	.0006	-.0001	.1073	.7577	72.8	.0005	.0009	.0662	.6067	-.0001	.0003	.0903	.4213
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0012	.0010	.3581	6.311	.0073	.0013	.5248	11.64	0	-.0017	-.0002	.2702	8.027	-.0007	-.0021	.2898	6.803
2n/ln(n)	44	.0014	.0018	.1595	.7807	.0008	.0023	.2005	.9504	36	-.0001	-.0006	.1104	.4546	-.0015	-.0025	.1369	.8511
4n/ln(n)	86	-.0003	-.0009	.0759	.5261	.0018	.0023	.1050	1.390	72.8	.0003	-.0001	.0676	.5657	-.0012	-.0002	.0900	.4940
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0161	-.0004	2.270	31.11	.0120	-.0056	1.101	21.25	0	-.0147	.0025	.3088	35.26	-.0331	-.0003	2.194	32.70
2n/ln(n)	44	.0018	.0036	.2134	2.046	-.0018	-.0023	.2420	1.467	36	.0016	.0007	.1596	1.633	-.0008	-.0008	.1716	.9693
4n/ln(n)	86	-.0010	-.0010	.0072	.4434	.0006	.0008	.1074	1.192	72.8	-.0017	-.0019	.0710	.3834	.0005	.0001	.0944	.7990
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0010	-.0024	.2050	.3917	-.0005	-.0002	.2161	.5507	0	-.0022	-.0012	.1293	.5303	.0001	.0012	.1388	.9557
2n/ln(n)	44	-.0011	-.0009	.1505	.5247	-.0005	-.0026	.1756	.5246	36	-.0018	-.0001	.1015	.7649	.0011	.0025	.1189	.6344
4n/ln(n)	86	.0004	.0012	.0742	.5203	-.0011	-.0006	.1014	.6118	72.8	.0007	.0007	.0658	.4424	-.0005	-.0011	.0860	.5333
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0032	.0018	.2648	1.223	.0074	-.0059	2.090	30.99	0	.0010	.0026	.1698	.9790	-.0055	.0002	.9374	24.97
2n/ln(n)	44	.0047	.0031	.1596	.4607	.0009	.0026	.1983	.5651	36	.0015	.0009	.1098	.5577	-.0001	.0007	.1356	.5735
4n/ln(n)	86	-.0002	-.0003	.0759	.3622	-.0017	-.0023	.1031	.7632	72.8	-.0011	-.0009	.0678	.4125	-.0005	.0007	.0873	.7973
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0007	.0032	.5203	9.642	.0065	.0064	3.042	41.54	0	-.0013	-.0021	.3470	8.182	.0109	.0030	2.170	38.54
2n/ln(n)	44	.0017	.0035	.1877	.7993	.0004	.0003	.1756	1.279	36	-.0002	-.0011	.1341	.6003	-.0002	-.0001	.1687	1.177
4n/ln(n)	86	.0009	.0007	.0785	.6177	-.0001	.0003	.1645	.8986	72.8	.0005	.0008	.0707	.5435	.0002	-.0001	.1453	.8988

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.7. B. Trim-by-$p(X)$ Estimator Properties (Asymmetric Z, Known $p(X)$, $n = 500$, 1000)

		$n = 500$									$n = 1000$							
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0014	-.0002	.0926	.7455	-.0007	-.0008	.1019	.5296	0	.0002	-.0003	.0648	.6205	-.0002	-.0009	.0715	.7205
2n/ln(n)	32	.0010	.0011	.0752	.5047	-.0006	-.0001	.0891	.3208	29	-.0003	-.0012	.0537	.6672	-.0004	-.0007	.0640	.7785
4n/ln(n)	64.4	.0001	.0001	.0535	.3641	-.0007	-.0005	.0717	1.100	58	.0001	-.0002	.0411	.5982	.0005	.0009	.0540	.5795
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0014	.0004	.1900	7.015	-.0030	.0013	.2326	8.186	0	-.0027	-.0012	.1451	7.558	.0004	.0001	.1684	7.898
2n/ln(n)	32	.0006	.0005	.0820	.3519	-.0006	-.0002	.1006	.6625	29	-.0005	-.0004	.0612	.4772	.0011	.0010	.0725	.5335
4n/ln(n)	64.4	-.0003	-.0003	.0552	.3566	.0012	.0013	.0724	.5000	58	-.0007	-.0006	.0428	.5979	.0004	.0006	.0549	.5875
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0029	-.0027	1.191	28.22	.0117	-.0029	1.904	33.16	0	.0005	-.0026	1.145	29.43	-.0018	.0009	.5197	19.98
2n/ln(n)	32	.0002	-.0020	.1267	1.395	-.0013	-.0017	.1339	.7393	29	-.0013	-.0014	.0985	.9017	.0001	.0012	.1009	.8472
4n/ln(n)	64.4	.0001	.0003	.0584	.4191	.0006	.0009	.0776	.6113	58	-.0002	-.0003	.0481	.6734	-.0001	.0002	.0602	.6265
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0007	.0001	.0933	.4514	-.0014	-.0024	.0969	.6288	0	.0013	.0016	.0647	.4647	-.0005	-.0002	.0692	1.110
2n/ln(n)	32	.0011	.0012	.0751	.5591	.0003	-.0003	.0851	.6069	29	.0009	.0005	.0534	.4272	-.0001	.0009	.0603	.7599
4n/ln(n)	64.4	.0004	-.0001	.0542	.6382	-.0003	-.0003	.0683	.5582	58	.0001	-.0005	.0415	.8788	-.0002	-.0004	.0514	.4135
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0001	-.0003	.1172	.4355	.0277	-.0003	1.628	33.02	0	-.0008	-.0017	.0844	.5924	-.0113	-.0010	1.988	34.66
2n/ln(n)	32	-.0004	-.0003	.0807	.4480	.0011	.0001	.1006	.5115	29	-.0007	-.0012	.0593	.6817	.0009	.0000	.0738	.5331
4n/ln(n)	64.4	-.0001	.0004	.0554	.4675	.0002	-.0004	.0704	.4783	58	.0003	.0007	.0429	.4195	-.0007	-.0007	.0531	.4469
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0004	.0003	.2595	8.186	.0052	.0037	.2821	38.24	0	.0003	.0001	.1712	5.128	-.0120	-.0070	2.032	37.77
2n/ln(n)	32	-.0012	-.0021	.0997	.4156	-.0013	-.0002	.1439	.6888	29	.0006	.0006	.0743	.4773	.0005	.0004	.0843	.6502
4n/ln(n)	64.4	.0006	.0008	.0598	.8380	.0001	.0003	.0698	.7702	58	.0005	.0007	.0474	.4763	-.0003	-.0001	.0606	.7800

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.8. Trim-by-$p(X)$ Rejection Frequencies (Asymmetric Z, Known $p(X)$

$n = 100$				$n = 250$			
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .049, .101	.010, .048, .101	.011, .049, .102	.25	.010, .050, .098	.009, .049, .101	.012, .051, .100
1	.016, .038, .060	.011, .052, .104	.011, .049, .097	1	.013, .027, .045	.011, .053, .099	.010, .049, .100
2	.006, .008, .010	.019, .052, .091	.010, .050, .103	2	.004, .005, .007	.014, .050, .097	.011, .050, .100
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .054, 103	.011, .053, .098	.013, .050, .098	.25	.010, .049, .102	.011, .050, .099	.010, .051, .098
1	.013, .024, .039	.012, .052, .100	.013, .052, .102	1	.020, .038, .062	.011, .049, .099	.011, .051, .101
2	.013, .019, .026	.017, .052, .089	.015, .053, .098	2	.002, .004, .005	.015, .053, .097	.011, .049, .102
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.008, .050, .103	.009, .051, .102	.010, .049, .098	.25	.010, .049, .098	.011, .048, .098	.010, .049, .101
1	.013, .049, .099	.010 .050, .100	.010, .051, .099	1	.013, .052, .099	.012, .053, .101	.009, .052, .101
2	.030, .045, .057	.023, .050, .095	.022, .050, .102	2	.019, .037, .059	.012, .050, .099	.012, .049, .099
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .050, .099	.010, .049, .104	.013, .052, .098	.25	.012, .052, .099	.010, .049, .098	.011, .050, .097
1	.004, .006, .007	.012, .049, .099	.012, .051, .101	1	.009, .012, .014	.011, .048, .097	.011, .049, .101
2	.002, .003, .004	.025, .042, .084	.009, .047, .098	2	.001, .002, .002	.023, .040, .089	.012, .052, .102

$n = 500$				$n = 1000$			
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .048, .098	.010, .051, .102	.010, .052, .099	.25	.011, .050, .097	.010, .050, .100	.010, .051, .103
1	.012, .039, .052	.011, .051, .102	.009, .049, .099	1	.011, .025, .043	.010, .051, .101	.011, .048, .099
2	.008, .011, .014	.016, .050, .097	.011, .050, .100	2	.007, .009, .013	.014, .051, .098	.010, .051, .102
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .048, .099	.010, .049, .101	.011, .052, .101	.25	.010, .051, .102	.010, .054, .104	.011, .051, .102
1	.012, .028, .049	.012, .052, .099	.010, .052, .104	1	.014, .030, .052	.011, .050, .097	.011, .053, .099
2	.003, .005, .005	.013, .049, .095	.011, .051, .102	2	.012, .018, .025	.012, .053, .099	.011, .051, .099
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .051, .102	.010, .050, .100	.012, .052, .100	.25	.011, .050, .099	.010, .049, .098	.009, .051, .098
1	.010, .050, .102	.009, .053, .096	.010, .053, .099	1	.011, .053, .099	.010, .051, .100	.010, .051, .101
2	.013, .031, .051	.011, .052, .099	.022, .049, .102	2	.018, .040, .069	.012, .051, .099	.009, .051, .102
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .051, .096	.011, .049, .011	.009, .052, .101	.25	.011, .050, .102	.009, .052, .100	.011, .050, .100
1	.003, .004, .006	.011, .049, .100	.011, .048, .099	1	.002, .003, .004	.012, .049, .098	.012, .054, .098
2	.000, .001, .002	.014, .042, .087	.012, .049, .099	2	.002, .002, .002	.015, .044, .091	.011, .052, .103

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The true propensity score $p(X)$ is used to compute Z. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

H.3 Estimated $p(X)$: Case 1 (scalar X, symmetric Z, constant used for estimation)

Table H.9. (a) Estimator Properties (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 100, 250$)

	$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0063	.0059	.2063	.5187	.0011	-.0058	.2230	.6671	0	.0014	.0051	.1277	.4851	-.0111	-.0071	.1376	.4750
TT-BC(Z)	1	.0050	.0055	.2065	.6157	.0002	-.0011	.2177	.5810	.4	.0009	.0041	.1276	.5847	-.0119	-.0112	.1337	.3561
TT(X,$k_n^{(x)}$)	43	.0098	.0070	.1535	.6230	.0027	.0001	.1839	.4579	36	-.0004	.0060	.1009	.8046	-.0073	-.0083	.1205	.9825
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0103	-0052	.3049	1.458	.0295	.0147	.5107	3.846	0	-.0040	.0072	.3366	4.763	.0036	.0021	.2637	1.658
TT-BC(Z)	1	-.,0035	-.0144	.2127	.8490	.0017	.0052	.2354	.6707	.4	.0018	.0073	.1425	.5726	-.0075	-.0071	.1709	.4033
TT(X,$k_n^{(x)}$)	43	-.0047	-.0089	.1697	.6339	.0021	-.0019	.2042	.9943	36	.0028	.0024	.1126	.5170	-.0052	-.0122	.1331	.6969
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0154	.0051	.4775	3.251	.0197	.0127	.5456	3.038	0	-.0036	.0034	.3808	3.435	-.0128	-.0094	.4841	3.734
TT-BC(Z)	1	-.0013	.0003	.1883	.6815	-0039	-.0110	.2479	.8391	.4	.0041	.0009	.1446	.4284	-.0015	.0047	.1820	.6565
TT(X,$k_n^{(x)}$)	43	.0028	.0008	.2288	1.312	.0063	-.0040	.2667	.8659	36	.0027	.0001	.1668	.8105	.0012	.0028	.1855	.5852

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0079	-.0055	.2113	.4562	-.0056	.0054	.2235	.5820	0	-.0013	-.0006	.1311	.3800	.0009	.0023	.1505	.6724
TT-BC(Z)	1	-.0065	-.0071	.2122	.5408	-.0016	-.0066	.2094	.6753	.4	-.0024	-.0038	.1318	.5421	.0024	.0046	.1421	.4152
TT(X,$k_n^{(x)}$)	43	-.0055	-.0046	.1578	.5782	-.0085	-.0076	.1911	.3933	36	-.0010	-.0044	.1043	.7730	.0003	.0059	.1239	.9536
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0016	-.0083	.2782	.8916	.0330	.0202	.4553	4.497	0	-.0016	-.0016	.1708	.8765	-.0053	-.0082	.3125	2.770
TT-BC(Z)	1	.0024	.0044	.2196	.4269	-.0047	.0011	.2107	.5265	.4	-.0031	-.0021	.1475	.7376	-.0053	-.0019	.1579	.5707
TT(X,$k_n^{(x)}$)	43	.0007	.0007	.1622	.3223	.0007	.0033	.1985	.5874	36	.0001	-.0022	.1120	.4428	-.0016	.0008	.1440	.6186
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0155	-.0059	.4807	2.507	-.0069	-.0130	.3949	1.979	0	.0020	.0061	.3217	2.597	.0026	-.0080	.4907	4.820
TT-BC(Z)	1	.0040	.0006	.3262	.5874	-.0059	-.0051	.1994	.4834	.4	.0030	.0033	.1576	.4616	.0013	.0043	.1634	.7353
TT(X,$k_n^{(x)}$)	43	-.0017	.0051	.2022	.6784	-.0104	-.0171	.2652	1.027	36	.0062	.0077	.1249	.7118	-.0009	.0004	.2029	1.529

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.9. (b) Estimator Properties (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 500, 1000$)

		$n=500$									$n=1000$							
		$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**					$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0034	.0035	.0913	.4284	.0050	.0077	.0951	.8212	0	.0003	.0017	.0625	.7087	.0027	.0059	.0665	.5776
TT-BC(Z)	.4	.0026	.0027	.0905	.6491	.0028	.0042	.0941	.5315	.2	.0001	.0021	.0625	.8645	.0022	.0027	.0647	.5080
TT(X,$k_n^{(x)}$)	32	.0001	-.0006	.0766	.5704	-.0006	.0026	.0836	.8371	29	.0013	.0003	.0515	.7762	.0018	.0034	.0628	.4381
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0019	-.0016	.1731	1.827	-.0077	-.0061	.2179	2.337	0	.0023	.0030	.1116	1.548	.0023	.0016	.1426	1.525
TT-BC(Z)	.4	.0019	.00431	.1027	.4236	.0024	-.0012	.1092	.5605	.2	.0014	.0013	.0738	.5749	.0017	.0039	.0853	.8710
TT(X,$k_n^{(x)}$)	32	-.0002	-.0022	.0831	.4161	.0009	-.0003	.0943	.4657	29	.0010	.0039	.0589	.5872	.0007	-.0009	.0711	.5429
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0129	.0127	.4479	6.004	.0182	.0049	.4374	4.798	0	-.0300	-.0066	.5554	7.851	-.0006	-.0013	.3201	3.733
TT-BC(Z)	.4	.0054	.0110	.0956	.8842	-.0011	.0021	.1406	.5133	.2	.0005	-.0012	.0761	.5584	.0016	.0036	.0985	.7235
TT(X,$k_n^{(x)}$)	32	.0070	.0086	.1262	.9977	.0031	-.0005	.1300	.6765	29	-.0018	.0020	.0927	.9138	.0005	-.0005	.0975	.4897
		$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0018	.0013	.0900	.4997	.0009	-.0027	.1007	.4904	0	.0001	.0005	.0651	.3891	.0002	-.0001	.0678	.5834
TT-BC(Z)	.4	.0020	.0018	.0899	.3388	.0007	.0002	.0932	.4708	.2	-.0001	-.0009	.0651	.5401	.0010	-.0017	.0647	.6322
TT(X,$k_n^{(x)}$)	32	.0031	.0029	.0737	.6200	.0010	.0017	.0837	.4946	29	.0008	.0014	.0529	.4443	-.0002	-.0032	.0589	.8182
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0019	.0064	.1197	.5135	.0119	.0065	.3483	5.480	0	-.0022	-.0049	.0828	.6004	.0059	.0078	.3393	5.995
TT-BC(Z)	1	.0001	-.0008	.0991	.5235	.0016	.0011	.1036	.4696	.2	-.0015	-.0036	.0720	.4435	.0017	.0015	.0819	.4350
TT(X,$k_n^{(x)}$)	43	-.0013	-.0035	.0814	.8216	.0027	.0043	.1013	.4394	29	-.0017	-.0001	.0579	.5551	.0014	.0012	.0724	.5242
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0134	-.0040	.6883	8.268	-.0055	-.0201	.4111	2.809	0	.0034	-.0025	.1646	1.666	.0032	-.0122	.4794	5.165
TT-BC(Z)	1	.0008	-.0033	.2234	.4787	-.0004	-.0009	.1888	.5951	.2	.0037	.0004	.0872	.6990	-.0010	-.0029	.1711	.8004
TT(X,$k_n^{(x)}$)	43	.0023	.0021	.0989	.5875	-.0090	-.0092	.2291	1.102	29	.0027	.0037	.0728	.4829	.0006	-.0007	.2228	1.433

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. “No Trim” is the untrimmed estimator $\tilde{\theta}_n$; “TT(Z)” is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and “TT–BC(Z)” is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. “TT(X)” is $\theta_n^{(tx)}$; and “TT(X,k)” is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.10. Rejection Frequencies (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation

	$n = 100$				$n = 250$		
			$(Y_0, Y_1, X, U) \sim$ **Normal**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .053, .109	.003, .048, .103	.010, .048, .101	.25	.006, .053, .103	.007, .044, .098	.013, .051, .096
1	.020, .052, .081	.012, .055, .097	.011, .051, .097	1	.007, .010, .019	.009, .053, .108	.011, .041, .089
2	.011, .028, .046	.011, .051, .104	.014, .044, .081	2	.018, .032, .050	.009, .048, .100	.013, .055, .086
			$(Y_0, Y_1, X, U) \sim$ **Laplace**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .056, .096	.007, .048, .103	.010, .053, .094	.25	.011, .056, .106	.011, .053, .100	.012, .051, .102
1	.012, .025, .036	.007, .045, .094	.010, .048, .089	1	.020, .038, .062	.009, .050, .103	.016, .055, .093
2	.025, .042, .057	.013, .053, .095	.014, .045, .091	2	.019, .037, .052	.015, .049, .094	.016, .055, .098
			$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .045, .104	.013, .048, .101	.010, .056, .111	.25	.012, .057, .098	.010, .051, .094	.013, .056, .100
1	.016, .053, .093	.007, .049, .108	.010, .057, .099	1	.010, .060, .097	.008, .048, .093	.011, .048, .110
2	.026, .041, .053	.009, .053, .096	.009, .056, .102	2	.021, .037, .064	.008, .056, .106	.014, .048, .098
			$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .050, .096	.010, .052, .100	.013, .050, .102	.25	.013, .057, .100	.008, .048, .104	.014, .055, .104
1	.011, .020, .033	.005, .047, .106	.014, .047, .100	1	.019, .034, .058	.011, .048, .105	.010, .042, .091
2	.022, .042, .066	.009, .056, .109	.015, .057, .105	2	.016, .031, .042	.009, .056, .101	.018, .048, .084

	$n = 500$				$n = 1000$		
			$(Y_0, Y_1, X, U) \sim$ **Normal**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .050, .107	.010, .055, .106	.018, .0541, .097	.25	.015, .049, .091	.013, .048, .090	.009, .044, .093
1	.015, .037, .066	.009, .053, .107	.009, .055, .096	1	.013, .038, .066	.009, .043, .100	.010, .054, .096
2	.023, .025, .034	.009, .049, .097	.016, .056, .091	2	.012, .021, .024	.009, .059, .109	.017, .051, .103
			$(Y_0, Y_1, X, U) \sim$ **Laplace**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .056, .103	.010, .054, .106	.010, .056, .104	.25	.006, .053, .096	.003, .047, .100	.008, .050, .113
1	.020, .035, .063	.015, .048, .094	.009, .051, .100	1	.012, .042, .071	.012, .038, .095	.010, .056, .103
2	.023, .042, .051	.014, .050, .096	.014, .051, .099	2	.026, .044, .057	.012, .057, .097	.009, .044, .097
			$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .045, .107	.011, .052, .105	.012, .055, .093	.25	.008, .053, .098	.005 .049, .100	.014, .048, .102
1	.011, .057, .103	.011, .050, .102	.015, .056, .104	1	.011, .052, .099	.006, .055, .100	.016, .045, .102
2	.004, .006, .007	.010, .047, .098	.013, .054, .107	2	.023, .050, .067	.009, .046, .086	.007, .064, .108
			$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**				
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.008, .052, .099	.006, .048, .093	.012, .054, .099	.25	.020, .052, .088	.015, .049, .093	.009, .049, .090
1	.012, .021, .024	.006, .050, .106	.015, .052, .096	1	.019, .023, .030	.009, .052, .100	.013, .045, .099
2	.023, .039, .062	.007, .054, .105	.014, .056, .105	2	.009, .019, .024	.008, .054, .103	.014, .045, .097

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$.

Table H.11. (a) Trim-by-$p(X)$ Estimator Properties (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 100$, 250)

	$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0019	.0014	.2033	.4219	-.0015	-.0015	.2230	1.119	0	-.0017	-.0032	.1294	.7484	-.0012	-.0012	.1434	.4545
2n/ln(n)	44	.0005	.0009	.1506	.5689	-.0023	-.0002	.1868	.6596	36	-.0010	-.0012	.1015	.5845	-.0011	-.0022	.1235	.9964
4n/ln(n)	86	-.0008	.0003	.0751	.7752	.0006	-.0001	.1073	.7577	72.8	.0005	.0009	.0662	.6067	-.0001	.0003	.0903	.4213
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0012	.0010	.3581	6.311	.0073	.0013	.5248	11.64	0	-.0017	-.0002	.2702	8.027	-.0007	-.0021	.2898	6.803
2n/ln(n)	44	.0014	.0018	.1595	.7807	.0008	.0023	.2005	.9504	36	-.0001	-.0006	.1104	.4546	-.0015	-.0025	.1369	.8511
4n/ln(n)	86	-.0003	-.0009	.0759	.5261	.0018	.0023	.1050	1.390	72.8	.0003	-.0001	.0676	.5657	-.0012	-.0002	.0900	.4940
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0161	-.0004	2.270	31.11	.0120	-.0056	1.101	21.25	0	-.0147	.0025	.3088	35.26	-.0331	-.0003	2.194	32.70
2n/ln(n)	44	.0018	.0036	.2134	2.046	-.0018	-.0023	.2420	1.467	36	.0016	.0007	.1596	1.633	-.0008	-.0008	.1716	.9693
4n/ln(n)	86	-.0010	-.0010	.0072	.4434	.0006	.0008	.1074	1.192	72.8	-.0017	-.0019	.0710	.3834	.0005	.0001	.0944	.7990
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0010	-.0024	.2050	.3917	-.0005	-.0002	.2161	.5507	0	-.0022	-.0012	.1293	.5303	.0001	.0012	.1388	.9557
2n/ln(n)	44	-.0011	-.0009	.1505	.5247	-.0005	-.0026	.1756	.5246	36	-.0018	-.0001	.1015	.7649	.0011	.0025	.1189	.6344
4n/ln(n)	86	.0004	.0012	.0742	.5203	-.0011	-.0006	.1014	.6118	72.8	.0007	.0007	.0658	.4424	-.0005	-.0011	.0860	.5333
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0032	.0018	.2648	1.223	.0074	-.0059	2.090	30.99	0	.0010	.0026	.1698	.9790	-.0055	.0002	.9374	24.97
2n/ln(n)	44	.0047	.0031	.1596	.4607	.0009	.0026	.1983	.5651	36	.0015	.0009	.1098	.5577	-.0001	.0007	.1356	.5735
4n/ln(n)	86	-.0002	-.0003	.0759	.3622	-.0017	-.0023	.1031	.7632	72.8	-.0011	-.0009	.0678	.4125	-.0005	.0007	.0873	.7973
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0007	.0032	.5203	9.642	.0065	.0064	3.042	41.54	0	-.0013	-.0021	.3470	8.182	.0109	.0030	2.170	38.54
2n/ln(n)	44	.0017	.0035	.1877	.7993	.0004	.0003	.1756	1.279	36	-.0002	-.0011	.1341	.6003	-.0002	-.0001	.1687	1.177
4n/ln(n)	86	.0009	.0007	.0785	.6177	-.0001	.0003	.1645	.8986	72.8	.0005	.0008	.0707	.5435	.0002	-.0001	.1453	.8988

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. . A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.11. (b) Trim-by-$p(X)$ Estimator Properties (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 500$, 1000)

	$n = 500$									$n = 1000$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0014	-.0002	.0926	.7455	-.0007	-.0008	.1019	.5296	0	.0002	-.0003	.0648	.6205	-.0002	-.0009	.0715	.7205
2n/ln(n)	32	.0010	.0011	.0752	.5047	-.0006	-.0001	.0891	.3208	29	-.0003	-.0012	.0537	.6672	-.0004	-.0007	.0640	.7785
4n/ln(n)	64.4	.0001	.0001	.0535	.3641	-.0007	-.0005	.0717	1.100	58	.0001	-.0002	.0411	.5982	.0005	.0009	.0540	.5795
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0014	.0004	.1900	7.015	-.0030	.0013	.2326	8.186	0	-.0027	-.0012	.1451	7.558	.0004	.0001	.1684	7.898
2n/ln(n)	32	.0006	.0005	.0820	.3519	-.0006	-.0002	.1006	.6625	29	-.0005	-.0004	.0612	.4772	.0011	.0010	.0725	.5335
4n/ln(n)	64.4	-.0003	-.0003	.0552	.3566	.0012	.0013	.0724	.5000	58	-.0007	-.0006	.0428	.5979	.0004	.0006	.0549	.5875
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0029	-.0027	1.191	28.22	.0117	-.0029	1.904	33.16	0	.0005	-.0026	1.145	29.43	-.0018	.0009	.5197	19.98
2n/ln(n)	32	.0002	-.0020	.1267	1.395	-.0013	-.0017	.1339	.7393	29	-.0013	-.0014	.0985	.9017	.0001	.0012	.1009	.8472
4n/ln(n)	64.4	.0001	.0003	.0584	.4191	.0006	.0009	.0776	.6113	58	-.0002	-.0003	.0481	.6734	-.0001	.0002	.0602	.6265
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0007	.0001	.0933	.4514	-.0014	-.0024	.0969	.6288	0	.0013	.0016	.0647	.4647	-.0005	-.0002	.0692	1.110
2n/ln(n)	32	.0011	.0012	.0751	.5591	.0003	-.0003	.0851	.6069	29	.0009	.0005	.0534	.4272	-.0001	.0009	.0603	.7599
4n/ln(n)	64.4	.0004	-.0001	.0542	.6382	-.0003	-.0003	.0683	.5582	58	.0001	-.0005	.0415	.8788	-.0002	-.0004	.0514	.4135
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0001	-.0003	.1172	.4355	.0277	-.0003	1.628	33.02	0	-.0008	-.0017	.0844	.5924	-.0113	-.0010	1.988	34.66
2n/ln(n)	32	-.0004	-.0003	.0807	.4480	.0011	.0001	.1006	.5115	29	-.0007	-.0012	.0593	.6817	.0009	.0000	.0738	.5331
4n/ln(n)	64.4	-.0001	.0004	.0554	.4675	.0002	-.0004	.0704	.4783	58	.0003	.0007	.0429	.4195	-.0007	-.0007	.0531	.4469
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0004	.0003	.2595	8.186	.0052	.0037	.2821	38.24	0	.0003	.0001	.1712	5.128	-.0120	-.0070	2.032	37.77
2n/ln(n)	32	-.0012	-.0021	.0997	.4156	-.0013	-.0002	.1439	.6888	29	.0006	.0006	.0743	.4773	.0005	.0004	.0843	.6502
4n/ln(n)	64.4	.0006	.0008	.0598	.8380	.0001	.0003	.0698	.7702	58	.0005	.0007	.0474	.4763	-.0003	-.0001	.0606	.7800

The treatment assignment is $D = I(\alpha + \beta X > U)$ with $\alpha = 0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.12. Trim-by-$p(X)$ Rejection Frequencies (Case 1: Symmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation

$n=100$				$n=250$			
$(Y_0,Y_1,X,U)\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .049, .101	.010, .048, .101	.011, .049, .102	.25	.010, .050, .098	.009, .049, .101	.012, .051, .100
1	.016, .038, .060	.011, .052, .104	.011, .049, .097	1	.013, .027, .045	.011, .053, .099	.010, .049, .100
2	.006, .008, .010	.019, .052, .091	.010, .050, .103	2	.004, .005, .007	.014, .050, .097	.011, .050, .100
$(Y_0,Y_1,X,U)\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .054, 103	.011, .053, .098	.013, .050, .098	.25	.010, .049, .102	.011, .050, .099	.010, .051, .098
1	.013, .024, .039	.012, .052, .100	.013, .052, .102	1	.020, .038, .062	.011, .049, .099	.011, .051, .101
2	.013, .019, .026	.017, .052, .089	.015, .053, .098	2	.002, .004, .005	.015, .053, .097	.011, .049, .102

$(Y_0,Y_1,X)\sim$ **Normal**, $U\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.008, .050, .103	.009, .051, .102	.010, .049, .098	.25	.010, .049, .098	.011, .048, .098	.010, .049, .101
1	.013, .049, .099	.010 .050, .100	.010, .051, .099	1	.013, .052, .099	.012, .053, .101	.009, .052, .101
2	.030, .045, .057	.023, .050, .095	.022, .050, .102	2	.019, .037, .059	.012, .050, .099	.012, .049, .099
$(Y_0,Y_1,X)\sim$ **Laplace**, $U\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .050, .099	.010, .049, .104	.013, .052, .098	.25	.012, .052, .099	.010, .049, .098	.011, .050, .097
1	.004, .006, .007	.012, .049, .099	.012, .051, .101	1	.009, .012, .014	.011, .048, .097	.011, .049, .101
2	.002, .003, .004	.025, .042, .084	.009, .047, .098	2	.001, .002, .002	.023, .040, .089	.012, .052, .102

$n=500$				$n=1000$			
$(Y_0,Y_1,X,U)\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .048, .098	.010, .051, .102	.010, .052, .099	.25	.011, .050, .097	.010, .050, .100	.010, .051, .103
1	.012, .039, .052	.011, .051, .102	.009, .049, .099	1	.011, .025, .043	.010, .051, .101	.011, .048, .099
2	.008, .011, .014	.016, .050, .097	.011, .050, .100	2	.007, .009, .013	.014, .051, .098	.010, .051, .102
$(Y_0,Y_1,X,U)\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .048, .099	.010, .049, .101	.011, .052, .101	.25	.010, .051, .102	.010, .054, .104	.011, .051, .102
1	.012, .028, .049	.012, .052, .099	.010, .052, .104	1	.014, .030, .052	.011, .050, .097	.011, .053, .099
2	.003, .005, .005	.013, .049, .095	.011, .051, .102	2	.012, .018, .025	.012, .053, .099	.011, .051, .099

$(Y_0,Y_1,X)\sim$ **Normal**, $U\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .051, .102	.010, .050, .100	.012, .052, .100	.25	.011, .050, .099	.010, .049, .098	.009, .051, .098
1	.010, .050, .102	.009, .053, .096	.010, .053, .099	1	.011, .053, .099	.010, .051, .100	.010, .051, .101
2	.013, .031, .051	.011, .052, .099	.022, .049, .102	2	.018, .040, .069	.012, .051, .099	.009, .051, .102
$(Y_0,Y_1,X)\sim$ **Laplace**, $U\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .051, .096	.011, .049, .011	.009, .052, .101	.25	.011, .050, .102	.009, .052, .100	.011, .050, .100
1	.003, .004, .006	.011, .049, .100	.011, .048, .099	1	.002, .003, .004	.012, .049, .098	.012, .054, .098
2	.000, .001, .002	.014, .042, .087	.012, .049, .099	2	.002, .002, .002	.015, .044, .091	.011, .052, .103

The treatment assignment is $D=I(\alpha+\beta X>U)$ with $\alpha=0$, hence Z has a symmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

H.4 Estimated $p(X)$: Case 2 (scalar X, asymmetric Z, constant used for estimation)

Table H.13. (a) Estimator Properties (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 100, 250$)

		$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**					$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**					$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$					$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$					$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0001	.0061	.2115	.5906		.0079	.0016	.2335	.5563	0	.0003	.0009	.1314	.6709		.0014	.0018	.1432	.3078
TT-BC(Z)	1	-.0011	-.0023	.2058	.3966		.0042	.0052	.2173	.5874	.4	.0003	.0007	.1286	.7109		.0032	-.0035	.1369	.6598
TT(X,$k_n^{(x)}$)	43	.0041	.0037	.1590	.4622		.0029	-.0011	.1867	.5114	36	-.0001	.0008	.1033	.4664		-.0029	-.0003	.1267	.5278
		$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0063	-.0106	.3125	.9973		-.0130	-.0025	.3829	1.789	0	-.0002	-.0050	.2562	2.453		.0117	.0117	.3383	3.501
TT-BC(Z)	1	.0019	.0012	.2232	.5420		-.0009	-.0028	.2594	.7591	.4	-.0006	-.0002	.1478	.4855		.0046	.0063	.1601	.4024
TT(X,$k_n^{(x)}$)	43	-.0037	-.0053	.1761	.5201		-.0066	-.0085	.2013	.5664	36	.0027	.0025	.1160	.7261		.0010	.0046	.1367	.5615
		$\beta = 2$ $(\kappa = 1.25)$					$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$					$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0102	-.0186	.4293	2.899		-.0042	.0012	.6373	3.600	0	.0146	.0037	.4078	3.970		.0049	.0216	.5564	4.727
TT-BC(Z)	1	.0086	.0062	.1842	.6886		-.0128	-.0114	.2321	.6602	.4	.0036	.0091	.2002	.5793		.0084	.0064	.1878	.7666
TT(X,$k_n^{(x)}$)	43	.0043	-.0114	.2277	1.511		.0006	.0030	.2876	1.063	36	.0039	.0044	.1686	.6969		.0027	.0033	.1753	.6598
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$					$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$					$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0040	-.080	.2143	.5586		-.0003	.0055	.2318	.7847	0	.0066	.0119	.1328	.5267		-.0018	-.0037	.1487	.5895
TT-BC(Z)	1	-.0061	-.0022	.2215	.5869		.0090	.0007	.2029	.5884	.4	.0060	.0095	.1313	.6302		-.0005	-.0009	.1368	.5835
TT(X,$k_n^{(x)}$)	43	-.0018	.0070	.1543	.8587		.0055	.0058	.1873	.4996	36	.0040	.0042	.1035	.2908		.0043	.0030	.1188	.5390
		$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0079	.0113	.2738	.8778		-.0014	.0079	.3585	1.410	0	.0074	.0071	.1775	.5125		.0116	-.0056	.3750	3.569
TT-BC(Z)	1	.0056	.0089	.2139	.5951		.0099	.0090	.2146	.6282	.4	.0055	-.0025	.1458	1.028		-.0002	.0000	.1595	.3209
TT(X,$k_n^{(x)}$)	43	.0022	.0057	.1667	.5445		.0118	.0139	.2123	.5083	36	.0039	.0040	.1113	.3661		-.0014	-.0033	.1495	.3318
		$\beta = 2$ $(\kappa = 1.25)$					$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$					$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0071	.0112	.5090	3.125		.0024	-.0101	.4549	3.063	0	.0070	-.0056	.3247	2.725		.0034	.0202	.4112	4.564
TT-BC(Z)	1	.0028	.0008	.2296	.8215		.0034	-.0031	.2027	.8432	.4	.0012	.0029	.1640	.5842		-.0008	-.0018	.1645	.6868
TT(X,$k_n^{(x)}$)	43	-.0010	.0001	.1946	.7856		.0003	-.0070	.2724	.9566	36	.0005	-.0005	.1336	.3248		-.0005	.0024	.2440	.5703

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.13. (b) Estimator Properties (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 500, 1000$)

	$n=500$										$n=1000$									
		$(Y_0,Y_1,X,U) \sim$ **Normal**					$(Y_0,Y_1,X,U) \sim$ **Laplace**					$(Y_0,Y_1,X,U) \sim$ **Normal**					$(Y_0,Y_1,X,U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0031	.0025	.0932	.6328		-.0070	-.0035	.1033	.5594	0	.0012	.0022	.0641	.5129		.0019	.0022	.0688	.5525
TT-BC(Z)	.4	.0031	-.0005	.0914	.5648		-.0074	-.0058	.1010	.7710	.2	.0016	.0018	.0637	.5922		.0025	.0022	.0671	.5145
TT(X,$k_n^{(x)}$)	32	.0008	.0028	.0763	.4326		-.0045	-.0034	.0920	.4190	29	.0017	.0025	.0520	.4835		.0007	.0028	.0625	.4432
		$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0001	-.0063	.2022	3.167		.0109	-.0082	.2655	3.510	0	-.0013	.0008	.1356	2.448		-.0068	.0036	.1497	1.543
TT-BC(Z)	.4	-.0021	-.0043	.0962	.4686		-.0051	-.0021	.1135	.7727	.2	.0001	.017	.0762	.4689		-.0022	-.0015	.0852	.2989
TT(X,$k_n^{(x)}$)	32	.0013	.0027	.0838	.3780		-.0028	-.0041	.0982	.5666	29	.0007	.0006	.0611	.3699		-.0006	-.0029	.0727	.6343
		$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0045	.0024	.7023	7.801		-.0074	.0120	.4846	5.287	0	.0052	.0081	.4498	6.532		-.0174	-.0148	.5012	6.808
TT-BC(Z)	.4	-.0023	-.0019	.1527	.7862		.0062	.0088	.1184	.7016	.2	-.0043	.0000	.1521	.7652		-.0021	-.0002	.0989	.5066
TT(X,$k_n^{(x)}$)	32	.0030	.0018	.1346	.8759		.0049	.0049	.1368	.9040	29	.0006	-.0008	.1003	1.035		-.0055	-.0016	.1007	.6632

		$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0044	.0039	.0961	.5766		.0014	-.0003	.1063	.7747	0	.0005	-.0001	.0666	.7087		.0042	.0078	.0741	.6366
TT-BC(Z)	.4	.0031	.0048	.0955	.4222		-.0024	-.0059	.0996	.5972	.2	.0009	.0016	.0662	.4961		.0023	.0012	.0680	.9675
TT(X,$k_n^{(x)}$)	32	.0028	.0058	.0778	.6936		-.0032	-.0092	.0901	.8598	29	.0021	.0016	.0562	.4643		.0052	.0041	.0625	.4073
		$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0012	.0031	.1256	.6601		-.0088	-.0059	.2474	2.716	0	-.0011	-.0016	.0846	.5371		-.0010	.0011	.2374	4.003
TT-BC(Z)	1	.0000	-.0017	.1039	.5891		-.0028	-.0043	.1046	.5973	.2	-.0014	.0003	.0735	.5068		.0033	.0025	.0833	.6809
TT(X,$k_n^{(x)}$)	43	.0014	.0015	.0834	.3996		-.0033	-.0013	.1018	.6175	29	-.0010	.0025	.0573	.6063		.0006	-.0030	.0758	.7033
		$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0020	-.0035	.2918	3.025		.0103	.0351	.5467	7.004	0	.0053	.0027	.1667	1.645		.0154	.0220	.4699	6.946
TT-BC(Z)	1	.0004	-.0003	.1162	.5258		.0080	.0008	.1409	.7333	.2	.0010	.0030	.0909	.5589		.0102	.0004	.1579	.9232
TT(X,$k_n^{(x)}$)	43	.0015	.0026	.1019	.5891		-.0050	-.0055	.1865	.8902	29	-.0024	-.0024	.0783	.5359		.0096	.0014	.1376	.6721

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.14. Rejection Frequencies (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation

$n = 100$					$n = 250$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .057, .106	.006, .046, .101	.008, .053, .103	.25	.009, .046, .098	.011, .045, .099	.011, .045, .099
1	.016, .044, .081	.010, .047, .094	.009, .049, .099	1	.016, .030, .056	.010, .052, .108	.014, .049, .100
2	.032, .043, .069	.009, .057, .102	.019, .046, .081	2	.018, .033, .043	.011, .048, .097	.016, .047, .085
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.013, .056, .093	.006, .045, .099	.009, .050, .106	.25	.008, .047, .106	.013, .049, .100	.006, .054, .105
1	.025, .044, .062	.011, .052, .098	.010, .050, .105	1	.012, .024, .035	.012, .050, .092	.009, .050, .097
2	.030, .039, .059	.015, .039, .092	.018, .044, .082	2	.023, .037, .045	.017, .058, .104	.015, .049, .092
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .049, .103	.008, .045, .093	.009, .051, .093	.25	.010, .045, .101	.007, .051, .098	.009, .050, .103
1	.018, .046, .092	.004, .050, .113	.007, .047, .095	1	.015, .050, .095	.006 ,045, .108	.009, .045, .096
2	.016, .028, .042	.010, .042, .085	.016, .056, .091	2	.016, .037, .057	.008 ,.048, .098	.009, .050, .098
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .047, .094	.008, .046, .103	.012, .051, .098	.25	.011, .046, .091	.008, .042, .104	.006, .045, .107
1	.023, .050, .081	.011, .053, .104	.012, .047, .104	1	.016, .027, .047	.009, .045, .095	.015, .046, .087
2	.022, .034, .045	.006, .044, .092	.005, .045, .094	2	.012, .026, .030	.005, .046, .095	.011, .049, .110

$n = 500$					$n = 1000$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .055, .106	.010, .060, .110	.012, .050, .092	.25	.012, .047, .105	.005, .052, .106	.012, .048, .094
1	.017, .030, .043	.007, .054, .104	.010, .057, .101	1	.012, .025, .044	.010, .057, .106	.009, .052, .096
2	.014, .018, .024	.008, .046, .092	.012, .041, .089	2	.016, .025, .034	.016, .054, .097	.019, .047, .086
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.017, .052, .099	.014, .053, .107	.008, .048, .107	.25	.009, .059, .105	.011, .056, .098	.015, .054, .092
1	.013, .020, .039	.008, .052, .096	.009, .049, .102	1	.020, .030, .038	.012, .039, .093	.016, .052, .092
2	.015, .024, .032	.009, .055, .103	.013, .048, .104	2	.007, .018, .024	.014, .046, .099	.011, .047, .101
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .049, .094	.006, .053, .096	.008, .054, .112	.25	.007, .048, .097	.006, .049, .097	.006, .043, .103
1	.016, .065, .093	.006, .051, .109	.010, .050, .106	1	.010, .051, .088	.007, .050, .103	.010, .049, .100
2	.022, .031, .053	.011, .053, .089	.010, .050, .097	2	.019, .049, .094	.011, .042, .099	.008, .054, .097
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.013, .040, .089	.007, .050, .097	.012, .043, .101	.25	.012, .047, .101	.011, .047, .093	.011, .046, .098
1	.020, .044, .064	.009, .048, .093	.010, .049, .105	1	.016, .034, .048	.015, .053, .103	.012, .054, .097
2	.014, .019, .029	.011, .052, .105	.010, .054, .108	2	.008, .012, .020	.006, .047, .096	.012, .047, .094

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$.

Table H.15. (a) Trim-by-$p(X)$ Estimator Properties (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 100$, 250)

	$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0001	.0061	.2115	.5908	-.0086	-.0067	.2367	.5201	0	.0059	.0067	.1319	.4583	.0047	.0100	.1437	.7732
2n/ln(n)	44	.0033	-.0021	.1568	.5361	-.0125	-.0168	.1910	.4446	36	.0032	.0031	.1031	.4526	.0061	.0094	.1256	.6056
4n/ln(n)	86	.0018	.0058	.0971	.6410	-.0005	-.0037	.1027	.5212	72.8	.0021	.0036	.0658	.7070	.0020	.0015	.0902	.7254
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0282	-.0047	.3574	2.472	-.0131	.0046	.4822	3.117	0	.0060	.0147	.1995	1.213	-.0152	-.0090	.2558	1.911
2n/ln(n)	44	.0008	.0060	.1654	.7337	.0091	.0064	.2074	.4402	36	.0048	.0048	.1127	.4049	-.0021	-.0012	.1411	.6968
4n/ln(n)	86	-.0022	-.036	.0771	.7502	.0021	.0002	.0989	.6261	72.8	-.0021	-.0042	.0691	.6906	.0043	.0012	.0899	.5043
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0033	.0074	.5223	3.700	.0523	.0307	.7496	4.397	0	.0361	-.0050	.6559	7.089	.0060	.0029	.6726	5.510
2n/ln(n)	44	.0091	-.0044	.2568	2.031	.0105	.0039	.2799	1.422	36	-.0026	-.0029	.1686	1.304	-.0008	.0023	.1894	1.316
4n/ln(n)	86	.0035	.0027	.0824	.7122	-.0034	-.0028	.1116	.9228	72.8	.0013	-.0016	.0721	.5779	-.0002	-.0027	.0948	.4281
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0093	.0226	.2105	.7777	.0013	.0016	.2334	.4900	0	.0047	.0051	.1366	.6173	.0009	-.0010	.1471	.3905
2n/ln(n)	44	.0010	.0013	.1554	.5987	.0018	-.0020	.1818	.5975	36	.0046	.0053	.1048	.4378	-.0036	-.015	.1210	.5914
4n/ln(n)	86	.0041	.0056	.0767	.3914	.0008	.0014	.1094	.7102	72.8	.0006	.0004	.0686	.7117	.0003	.0002	.0877	.7413
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0039	-.0052	.2791	.7772	-.0057	-.0035	.3936	2.074	0	-.0120	-.0165	.1751	.6002	.0001	.0038	.2924	2.011
2n/ln(n)	44	-.0069	-.0081	.1637	.3941	-.0033	.0019	.2127	.6772	36	-.0112	-.0112	.1156	.5254	-.0001	.0060	.1397	.7473
4n/ln(n)	86	.0005	.0008	.0793	.5402	-.0079	-.0090	.1051	.5314	72.8	-.0004	-.0042	.0682	.7950	-.0013	-.0012	.0903	.4662
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0049	.0121	.5564	3.447	-.0042	-.0062	.4379	5.230	0	.0036	.0103	.3993	3.918	.0242	.0087	.5921	6.221
2n/ln(n)	44	.0137	.0134	.2071	.8680	.0020	.0026	.2409	.7990	36	.0055	.0067	.1373	.4956	.0014	.0009	.1967	.9921
4n/ln(n)	86	.0010	.0023	.0806	.8191	.0006	-.0002	.2097	.6172	72.8	-.0059	-.0043	.0723	.5193	-.0009	-.0012	.1782	.8882

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. . A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.15. (b) Trim-by-$p(X)$ Estimator Properties (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation, $n = 500, 1000$)

		$n = 500$										$n = 1000$						
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**						$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**		
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)						$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0030	.0050	.0949	.4339	-.0041	-.0026	.1040	.6225	0	.0019	.0031	.0645	.5112	.0045	.0042	.0721	.3706
2n/ln(n)	32	.0007	-.0006	.0763	.5043	-.0016	-.0025	.0902	.3579	29	.0008	-.0007	.0555	.5064	.0032	.0046	.0655	.5163
4n/ln(n)	64.4	-.0033	-.0056	.0547	.7242	.0029	.0023	.0700	.5360	59	-.0006	-.0006	.0428	.5484	-.0014	-.0023	.0551	.4305
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)						$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0096	.0093	.1756	1.396	-.0020	-.0069	.2517	2.978	0	.0011	.0052	.1133	1.495	-.0054	-.0021	.1359	1.131
2n/ln(n)	32	.0026	-.0002	.0858	.6810	.0006	.0022	.1024	.6125	29	-.0007	-.0007	.0601	.3533	.0002	-.0016	.0715	.5311
4n/ln(n)	64.4	.0026	.0031	.0546	.4999	.0021	.0047	.0730	.7454	59	.0007	-.0003	.0446	.5096	-.0003	.0014	.0534	.7226
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)						$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0245	.0081	.7835	7.967	-.0101	.0153	.5944	6.956	0	-.0096	.0109	.4824	6.915	.0364	.0075	.9661	9.304
2n/ln(n)	32	.0087	.0068	.1385	1.089	.0057	.0032	.1280	.4323	29	.0013	.0025	.1032	.8475	.0001	.0031	.0976	.7029
4n/ln(n)	64.4	-.0052	-.0058	.0657	.5218	-.0010	.0040	.0772	.9760	59	-.0005	-.0005	.0493	.5261	.0008	-.0007	.0589	.5790
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**						$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**		
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)						$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0017	.0038	.0930	.9582	.0046	.0075	.1017	.5770	0	-.0025	-.0015	.0673	.7609	.0012	.0004	.0751	.5527
2n/ln(n)	32	.0015	-.0014	.0751	.7085	.0014	.0033	.0884	.6820	29	-.0007	.0009	.0561	.4932	.0011	.0023	.0654	.4282
4n/ln(n)	64.4	.0009	.0031	.0552	.6928	-.0010	-.0070	.0743	.5483	59	.0017	.0022	.0417	.4513	.0008	-.0006	.0517	.6507
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)						$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0061	-.0046	.1223	.4603	.0089	.0082	.6026	7.285	0	-.0018	-.0050	.0836	.6504	-.0082	-.0066	.3205	4.802
2n/ln(n)	32	-.0033	-.0044	.0843	.4678	.0036	-.0012	.1043	.8408	29	.0004	-.0013	.0596	.4158	.0003	.0024	.0770	.4224
4n/ln(n)	64.4	-.0010	-.0009	.0561	.4832	.0010	.0002	.0711	.4981	59	.0001	-.0008	.0423	.7224	-.0033	-.0010	.0548	.5739
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)						$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)		
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0074	.0053	.2280	2.138	.0029	.0017	.5192	5.542	0	-.0038	-.0001	.1838	2.344	-.0130	-.0244	.4499	7.190
2n/ln(n)	32	.0037	.0017	.0997	.5712	.0022	.0016	.1987	1.102	29	.0025	.0017	.0751	.6628	-.0011	.0004	.1019	.8970
4n/ln(n)	64.4	.0019	.0011	.0583	.8070	-.0012	.0007	.1221	.8282	59	-.0011	-.0015	.0485	.5471	.0017	.0016	.0962	.6274

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.16. Trim-by-$p(X)$ Rejection Frequencies (Case 2: Asymmetric Z, Estimated $p(X)$, X is scalar, constant term used in estimation

$n = 100$				$n = 250$			
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .057, .106	.009, .049, .113	.011, .045, .100	.25	.007, .050, .104	.010, .054, .105	.011, .050, .104
1	.016, .033, .058	.015, .054, .099	.010, .047, .108	1	.019, .043, .066	.006, .051, .102	.015, .053, .100
2	.022, .033, .044	.010, .035, .057	.014, .047, .088	2	.012, .017, .025	.019, .050, .085	.011, .050, .106
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.007, .040, .085	.008, .051, .100	.014, .052, .103	.25	.015, .044, .092	.009, .051, .091	.014, .054, .108
1	.017, .026, .044	.013, .052, .101	.014, .051, .095	1	.019, .042, .079	.011, .044, .091	.013, .051, .103
2	.027, .044, .058	.013, .041, .073	.017, .059, .101	2	.017, .027, .034	.009, .041, .083	.008, .048, .102
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.013, .054, .099	.015, .055, .099	.010, .044, .101	.25	.007, .042, .104	.008, .049, .099	.010, .048, .093
1	.016, .043, .086	.014, .045, .103	.012, .048, .102	1	.011, .047, .088	.010, .043, .103	.005, .047, .103
2	.018, .032, .049	.021, .056, .084	.013, .045, .104	2	.010, .016, .028	.014, .056, .096	.009, .045, .098
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.018, .046, .087	.018, .047, .095	.015, .057, .099	.25	.009, .046, .103	.011, .054, .106	.008, .053, .101
1	.017, .042, .074	.013, .052, .098	.011, .055, .096	1	.026, .044, .067	.018, .053, .100	.013, .037, .102
2	.011, .038, .055	.015, .042, .088	.007, .047, .097	2	.005, .021, .056	.017, .044, .083	.014, .047, .097

$n = 500$				$n = 1000$			
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.006, .046, .101	.010, .054, .097	.009, .051, .095	.25	.009, .050, .104	.011, .046, .101	.009, .058, .096
1	.023, .043, .076	.010, .051, .109	.008, .055, .100	1	.022, .050, .082	.011, .047, .098	.010, .049, .099
2	.007, .015, .015	.020, .048, .076	.008, .053, .091	2	.014, .020, .031	.014, .045, .096	.007, .047, .096
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .063, .101	.011, .049, .095	.015, .051, .102	.25	.014, .055, .090	.011, .044, .101	.010, .042, .095
1	.010, .022, .038	.014, .054, .095	.013, .044, .090	1	.022, .049, .087	.017, .048, .095	.010, .047, .108
2	.016, .025, .031	.013, .047, .090	.008, .050, .093	2	.005, .005, .007	.013, .046, .098	.005, .048, .097
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.008, .053, .101	.009, .052, .096	.009, .052, .112	.25	.011, .053, .108	.008, .044, .099	.009, .049, .101
1	.014, .049, .087	.007, .051, .095	.011, .048, .105	1	.014, .048, .099	.008, .055, .100	.011, .040, .095
2	.024, .047, .072	.011, .048, .094	.005, .049, .096	2	.018, .045, .067	.009, .043, .092	.006, .049, .105
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .049, .089	.008, .039, .098	.008, .047, .100	.25	.011, .057, .100	.009, .050, .098	.009, .054, .107
1	.009, .010, .011	.016, .055, .113	.015, .054, .097	1	.013, .023, .036	.013, .051, .091	.007, .046, .101
2	.009, .009, .010	.014, .058, .109	.014, .055, .097	2	.008, .012, .015	.015, .054, .110	.009, .052, .106

The treatment assignment is $D = I(.25 + \beta X > U)$, hence Z has an asymmetric distribution. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

H.5 Estimated $p(X)$: Case 3 (multivariate X, symmetric Z, constant used for estimation)

Table H.17. (a) Estimator Properties (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 100$, 250)

		$n = 100$									$n = 250$							
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0037	-.0108	.2155	.4913	.0037	.0047	.2717	.8344	0	.0001	.0041	.1389	.6342	-.0033	.0042	.1836	1.403
TT-BC(Z)	1	-.0023	.0036	.1977	.6097	.0057	.0138	.2144	.7226	.4	.0022	.0050	.1325	.3546	-.0038	.0011	.1393	.6782
TT(X,$k_n^{(x)}$)	43	.0018	-.0001	.1503	.7479	.0090	.0059	.1938	.4769	36	.0034	.0094	.1026	.8306	.0005	-.0003	.1250	.3928
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0074	-.0066	.2143	.5041	.0113	.0113	.2789	1.300	0	-.0037	.0004	.1716	1.800	.0026	.032	.1888	1.172
TT-BC(Z)	1	.0023	-.0002	.1931	.6094	.0074	.0104	.2096	.4800	.4	-.0022	-.0001	.1254	.5913	.0014	-.0048	.1353	.7882
TT(X,$k_n^{(x)}$)	43	-.0039	-.0065	.1503	.6693	.0079	.0048	.1856	.9109	36	-.0019	-.0014	.1006	.5319	.0020	-.0001	.1266	.7062
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0008	.0044	.2417	.7380	-.0032	-.0068	.3318	1.925	0	-.0004	-.0046	.2427	3.753	-.0077	-.0049	.3059	4.185
TT-BC(Z)	1	.0034	-.0023	.2010	.5181	.0018	.0051	.1996	.5582	.4	-.0016	-.0009	.1278	.5723	-.0087	-.0065	.1405	.4332
TT(X,$k_n^{(x)}$)	43	.0037	.0049	.1711	.5776	.0061	.0003	.1931	.4946	36	.0014	.0012	.1031	.4790	-.0007	.0004	.1280	.4626

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$					$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0033	.0012	.2207	.3956	-.0027	-.0075	.2582	.4617	0	.0002	.0031	.1362	.5573	-.0033	.0021	.1915	1.582
TT-BC(Z)	1	.0032	.0016	.2092	.3844	.0008	-.0004	.2160	.4652	.4	.0009	.0046	.1307	.4030	-.0002	-.0041	.1382	.4370
TT(X,$k_n^{(x)}$)	43	.0062	.0102	.1559	.4897	-.0045	-.0049	.1909	.5084	36	.0040	.0052	.1051	.7283	-.0018	-.0020	.1221	.5189
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$					$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0032	-.0031	.2582	1.313	-.0065	-.0039	.2609	1.200	0	-.0070	-.0041	.1514	.6261	.0070	.0061	.1846	1.657
TT-BC(Z)	1	-.0046	-.0001	.2095	.6407	-.0042	.0012	.2119	.4949	.4	-.0086	-.0011	.1329	.8375	.0039	.0000	.1353	.7565
TT(X,$k_n^{(x)}$)	43	.0001	.0017	.1593	.5754	-.0020	-.0081	.1934	.6497	36	-.0061	-.0034	.1068	.4959	.0024	.0026	.1237	.4657
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$					$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0064	.0001	.2746	1.494	-.0072	-.0031	.3333	2.432	0	-.0074	-.0065	.1754	.8329	.0176	-.0022	.3792	3.299
TT-BC(Z)	1	.0044	.0091	.1935	.4777	-.0031	.0003	.1992	.5450	.4	-.0025	-.0004	.1391	.4918	.0006	-.0003	.1309	.8812
TT(X,$k_n^{(x)}$)	43	.0033	.0069	.1599	.6204	-.0006	.0013	.1848	.7871	36	-.0022	-.0015	.1113	.6317	-.0011	-.0001	.1087	.7002

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.17. (b) Estimator Properties (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 500, 1000$)

	$n=500$										$n=1000$							
		$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**					$(Y_0,Y_1,X,U) \sim$ **Normal**				$(Y_0,Y_1,X,U) \sim$ **Laplace**			
		$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$					$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0007	-.0027	.1001	.5566	-.0005	-.0045	.1203	.6388	0	-.0005	-.0018	.0708	.3975	-.0017	-.0017	.1249	3.361
TT-BC(Z)	.4	.0001	.0014	.0929	.3931	-.0015	-.0008	.1004	.5553	.2	-.0009	-.0025	.0671	.5237	-.0002	-.0003	.0689	.4875
TT(X,$k_n^{(x)}$)	32	.0031	.0051	.0729	.5486	.0003	.0025	.0906	.5672	29	-.0005	-.0015	.0542	.4621	.0013	.0004	.0626	.9456
		$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$					$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0009	.0021	.1169	1.101	.0013	.0008	.1655	2.536	0	-.0004	-.0012	.0857	1.445	-.0012	.0043	.1617	3.775
TT-BC(Z)	.4	-.0007	-.0001	.0911	.4222	.0020	.0031	.0970	.5059	.2	.0022	.00321	.0661	.4844	.0012	.0007	.0726	.5596
TT(X,$k_n^{(x)}$)	32	-.0032	-.0007	.0756	.5754	.0057	.0023	.0876	.8891	29	.0003	.0024	.0553	.5757	.0018	.0009	.0657	.5116
		$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$					$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0012	.0006	.1343	1.542	.0064	.0015	.5325	8.718	0	.0043	.0009	.1655	4.810	-.0019	.0040	.3739	7.695
TT-BC(Z)	.4	.0033	.0045	.0868	.6686	-.0009	.0000	.0958	.6551	.2	-.0021	-.0045	.0675	.7476	-.0013	.0005	.0712	.5319
TT(X,$k_n^{(x)}$)	32	.0029	.0045	.0760	.9112	-.0016	.0036	.0955	.6931	29	-.0005	-.0010	.0586	.7244	-.0015	-.0028	.0697	.7238

		$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0,Y_1,X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0,Y_1,X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$					$\beta=.25$ $(\kappa=17)$				$\beta=.25$ $(\kappa=5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0005	.0012	.0976	.5227	-.0045	-.0044	.1915	4.141	0	.0038	.0016	.0684	.6118	-.0049	.0012	.1913	6.061
TT-BC(Z)	.4	-.0008	.0030	.0954	.6681	.0022	-.0026	.0941	.7102	.2	.0027	.0032	.0665	.7605	.0001	.0010	.0689	.7099
TT(X,$k_n^{(x)}$)	32	-.0001	.0009	.0771	.4973	.0006	.0004	.0869	.5799	29	.0018	.0010	.0566	.4600	.0032	.0046	.0618	.5934
		$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$					$\beta=1$ $(\kappa=2)$				$\beta=1$ $(\kappa=2)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0044	-.0025	.1038	.5892	-.0069	-.0141	.1335	1.526	0	.0016	.0004	.0750	.6883	.0043	.0056	.2102	4.660
TT-BC(Z)	1	-.0027	-.0014	.0950	.4164	-.0040	-.0043	.0923	.3961	.2	.0002	-.0010	.0683	.4781	-.0004	-.0016	.0871	.6703
TT(X,$k_n^{(x)}$)	43	-.0032	-.0005	.0785	.8873	-.0031	-.0047	.0895	.5367	29	.0023	.0017	.0567	.5862	.0005	.0024	.0792	.7700
		$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$					$\beta=2$ $(\kappa=1.25)$				$\beta=2$ $(\kappa=1.5)$			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0046	-.0041	.1176	.7575	.0254	-.0055	.4635	5.552	0	-.0025	-.0030	.0928	1.001	-.0331	-.0052	.3938	6.722
TT-BC(Z)	1	-.0025	-.0001	.0893	.6272	.0008	.0021	.1944	.8833	.2	.0019	.0027	.0705	.4064	-.0011	-.0011	.1543	.7454
TT(X,$k_n^{(x)}$)	43	-.0006	-.0039	.0761	.6913	.0005	.0015	.1837	.7952	29	.0012	.0030	.0573	.6808	-.0007	.0004	.1502	.8662

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.18. Rejection Frequencies (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation

$n = 100$					$n = 250$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .046, .103	.005, .044, .097	.011, .047, .102	.25	.010, .054, .098	.010, .055, .100	.011, .056, .092
1	.012, .047, .102	.008, .046, .101	.011, .048, .103	1	.004, .016, .053	.005, .053, .106	.009, .048, .104
2	.012, .049, .089	.009, .053, .105	.011, .063, .107	2	.010, .107, .025	.010, .049, .091	.011, .055, .104
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .032, .073	.008, .053, .101	.013, .048, .103	.25	.008, .037, .062	.010, .055, .102	.009, .049, .099
1	.016, .037, .073	.010, .044, .099	.010, .052, .103	1	.016, .046, .080	.007, .051, .100	.009, .045, .086
2	.013, .028, .048	.006, .056, .103	.014, .049, .100	2	.017, .026, .034	.007, .052, .110	.007, .051, .102
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.009, .052, .094	.007, .055, .102	.008, .047, .105	.25	.010, .051, .101	.009, .051, .103	.015, .051, .097
1	.016, .042, .082	.015, .051, .107	.011, .054, .086	1	.014, .051, .088	.009, .051, .106	.009, .049, .096
2	.011, .032, .056	.013, .046, .100	.012, .050, .101	2	.018, .049, .088	.011, .046, .100	.010, .049, .094
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.013, .046, .096	.009, .049, .108	.014, .055, .103	.25	.010, .037, .070	.010, .051, .103	.014, .048, .102
1	.018, .045, .083	.014, .054, .105	.013, .050, .096	1	.020, .049, .072	.011, .048, .095	.014, .042, .096
2	.010, .023, .053	.012, .053, .096	.014, .045, .089	2	.006, .008, .014	.010, .056, .107	.007, .047, .095

$n = 500$					$n = 1000$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.008, .058, .114	.005, .053, .104	.011, .054, .105	.25	.010, .051, .092	.004, .056, .095	.009, .059, .107
1	.013, .042, .081	.005, .040, .106	.009, .052, .103	1	.012, .040, .083	.006, .042, .103	.015, .038, .085
2	.021, .044, .073	.008, .056, .102	.008, .051, .099	2	.008, .015, .024	.006, .038, .107	.006, .039, .107
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.015, .045, .090	.009, .054, .116	.009, .049, .088	.25	.006, .012, .020	.011, .047, .093	.011, .044, .095
1	.016, .031, .046	.005, .055, .103	.014, .058, .102	1	.006, .015, .026	.010, .049, .109	.008, .046, .106
2	.002, .005, .008	.009, .050, .106	.005, .053, .103	2	.013, .016, .019	.007, .046, .094	.,007, .053, .096
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.009, .052, .107	.012, .045, .098	.010, .050, .106	.25	.008, .055, .107	.008, .058, .099	.016, .052, .100
1	.012, .049, .104	.004, .049, .105	.006, .049, .098	1	.015, .048, .086	.010, .051, .099	.010, .036, .086
2	.014, .056, .104	.010, .057, .108	.010, .046, .094	2	.013, .040, .076	.010, .052, .101	.018, .053, .087
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.004, .007, .013	.004, .050, .102	.013, .054, .108	.25	.004, .006, .007	.006, .050, .092	.008, .051, .099
1	.016, .049, .070	.006, .046, .102	.011, .041, .095	1	.013, .024, .045	.007, .054, .107	.012, .054, .102
2	.004, .005, .006	.006, .047, .096	.011, .054, .093	2	.002, .003, .003	.011, .053, .106	.012, .050, .102

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$.

Table H.19. (a) Trim-by-$p(X)$ Estimator Properties (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, n = 100, 250)

$n = 100$

		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0037	-.0108	.2155	.4931	-.0097	-.0072	.2784	1.423
2n/ln(n)	44	.0031	.0057	.1497	.5966	.0078	.0037	.1896	.5101
4n/ln(n)	86	-.0042	-.0044	.0748	.6807	-.0030	-.0057	.1016	.5612
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0062	.0099	.2330	.6663	-.0038	-.0009	.2782	.9504
2n/ln(n)	44	.0005	-.0037	.1600	.5203	.0001	.0038	.1974	.5361
4n/ln(n)	86	.0022	.0023	.0782	.6506	-.0007	-.0010	.1048	.6478
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0094	.0010	.2464	.8514	-.0020	.0019	.3282	2.131
2n/ln(n)	44	-.0045	-.0071	.1674	.7129	.0085	.0085	.2223	.8258
4n/ln(n)	86	-.0009	-.0039	.0777	.5883	.0012	-.0020	.1114	.4686
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0027	-.0064	.2191	.4786	.0053	-.0009	.2484	.9603
2n/ln(n)	44	.0011	-.0019	.1557	.5335	-.0093	-.0064	.1873	.5062
4n/ln(n)	86	-.0023	-.0005	.0761	.6894	.0047	.0057	.1047	.5111
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0045	.0045	.2347	.6103	.0162	-.0011	.2940	1.011
2n/ln(n)	44	-.0015	.0009	.1553	.4074	-.0006	-.0145	.1923	.8693
4n/ln(n)	86	-.0001	.0005	.0786	.3712	.0023	.0026	.1108	.5617
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0146	-.0192	.2587	.8861	.0021	.0025	.2999	2.093
2n/ln(n)	44	-.0032	-.0064	.1650	.5489	-.0201	.0002	.1502	1.023
4n/ln(n)	86	-.0023	-.0034	.0815	.6462	-.0020	-.0018	.1226	.8992

$n = 250$

		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0037	.0001	.1418	.6485	.0071	.0006	.1647	.5272
2n/ln(n)	36	-.0002	.0040	.1037	.8038	.0030	.0021	.1212	.7667
4n/ln(n)	72.8	.0026	.0004	.0663	.0744	-.0019	-.0032	.0851	.5667
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim		.0046	.0009	.1716	1.335	-.0047	-.0086	.2471	3.003
2n/ln(n)		.0003	.0008	.1005	.5555	.0019	-.0005	.1301	.6608
4n/ln(n)		-.0004	.005	.0692	.5500	-.0023	.0030	.0937	.5835
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0097	-.0012	.1953	1.501	-.0152	-.0018	.2719	3.463
2n/ln(n)	36	-.0034	-.0012	.1155	.5892	-.0017	-.0050	.1384	.4433
4n/ln(n)	72.8	-.0007	.0010	.0694	.4720	-.0049	-.0061	.0940	.6276
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0017	-.0036	.1322	1.047	-.0016	-.0032	.1604	.6086
2n/ln(n)	36	.0028	.0005	.1016	.6267	-.0021	-.0009	.1204	.3908
4n/ln(n)	72.8	-.0004	-.0006	.0647	.5013	.0013	.0038	.0874	.5295
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0067	-.0128	.1478	.5794	-.0044	-.0083	.3029	1.542
2n/ln(n)	36	-.0022	-.0021	.1100	.5469	-.0003	.0004	.1758	.9304
4n/ln(n)	72.8	-.007	-.0008	.0681	.4962	.0022	.0005	.1442	.7534
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0148	-.0037	.1853	1.057	-.0065	-.0120	.4452	4.902
2n/ln(n)	36	-.0059	-.0014	.1103	.6466	.0002	.0000	.2201	1.221
4n/ln(n)	72.8	.0019	.0018	.0702	.5522	-.0002	-.0031	.1206	.7772

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. . A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.19. (b) Trim-by-$p(X)$ Estimator Properties (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, n $= 500, 1000$)

		$n = 500$								$n = 1000$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0020	.0019	.1014	.4601	.0025	.0024	.1131	.6850	0	-.0002	.0027	.0726	.5695	.0027	.0019	.0902	1.089
2n/ln(n)	32	-.0012	-.0021	.0757	.6179	.0049	.0058	.0899	.7945	29	.0004	.0005	.0565	.6214	-.0008	-.0010	.0604	.6045
4n/ln(n)	64.4	.0080	.0002	.0540	.4355	-.0024	-.0045	.0669	.4048	59	.0005	-.0005	.0417	.4728	-.0023	-.0033	.0511	.4548
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0028	-.0001	.1166	1.193	-.0013	-.0042	.1726	2.354	0	.0034	.0032	.0895	1.456	.0001	-.0033	.1790	4.697
2n/ln(n)	32	.0034	.0021	.0775	.4218	.0007	.0029	.0948	.6359	29	.0019	.0031	.0533	.3790	-.0020	-.0016	.0654	.6701
4n/ln(n)	64.4	-.0004	-.0019	.0556	.6814	-.0021	-.0052	.0704	.5346	59	.0023	.0018	.0420	.5839	-.0006	-.0022	.0532	.7444
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0031	-.0061	.1703	2.991	.0026	.0025	.2248	3.332	0	-.0019	.0020	.1395	3.231	-.0064	-.0042	.2346	4.672
2n/ln(n)	32	.0004	.0002	.0833	.4634	.0021	.0057	.1038	.4747	29	-.0006	-.0024	.0633	.5947	-.0037	-.0032	.0791	.6510
4n/ln(n)	64.4	.0015	.0022	.0569	.6121	.0035	.0025	.0749	.5361	59	-.0004	-.0011	.0427	.4813	-.0009	.0001	.0569	.4561
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0046	.0052	.0951	.4694	.0036	.0053	.1246	.8840	0	-.0013	-.0020	.0692	.6037	.0041	-.0009	.1231	2.606
2n/ln(n)	32	.0019	.0024	.0775	.6552	.0015	.0044	.0896	.6053	29	.0008	.0013	.0542	.6112	-.0001	-.0025	.0638	.5108
4n/ln(n)	64.4	.0002	.0014	.0531	.6618	.0001	-.0001	.0700	.3594	59	.0022	.0008	.0419	.6410	-.0008	-.0019	.0496	.5266
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0066	-.0117	.1045	.6218	.0177	.0085	.3392	2.652	0	-.0036	-.0031	.0743	.7737	.0052	.0049	.2986	1.921
2n/ln(n)	32	-.0028	-.0027	.0776	.4976	.0203	.0062	.2310	1.322	29	-.0022	-.0028	.0534	.5582	-.0103	.0008	.1501	1.022
4n/ln(n)	64.4	-.0026	-.0013	.0539	.5421	.0003	-.0006	.1540	.8920	59	.0006	.0012	.0405	.5915	.0015	.0007	.1033	.5672
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0003	.0032	.1398	1.760	.0045	.0109	.4429	5.502	0	-.0016	-.0001	.0986	2.129	-.0152	-.0062	.3992	4.902
2n/ln(n)	32	.0023	.0019	.0816	.6480	-.0109	-.0001	.1743	1.221	29	.0013	.0016	.0591	.6912	-.0004	.0042	.1652	1.221
4n/ln(n)	64.4	.002	-.0023	.0559	.7726	-.0002	-.0004	.1212	.6658	59	.0020	.0005	.0432	.5739	.0002	-.0004	.1034	.8842

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.20. Trim-by-$p(X)$ Rejection Frequencies (Case 3: Symmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation

	$n = 100$				$n = 250$		
			$(Y_0, Y_1, X, U) \sim$ **Normal**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.012, .046, .103	.009, .043, .098	.015, .053, .099	.25	.008, .048, .110	.011, .052, .103	.012, .057, .109
1	.014, .055, .091	.012, .055, .100	.013, .059, .109	1	.010, .036, .062	.011, .053, .101	.015, .050, .102
2	.017, .046, .085	.013, .053, .096	.011, .053, .103	2	.016, .036, .060	.010, .047, .101	.008, .043, .102
			$(Y_0, Y_1, X, U) \sim$ **Laplace**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .041, .077	.009, .059, .100	.016, .050, .091	.25	.015, .046, .090	.009, .054, .103	.011, .047, .105
1	.019, .052, .097	.011, .052, .100	.019, .054, .102	1	.017, .025, .044	.007, .051, .106	.008, .047, .103
2	.018, .031, .058	.018, .057, .101	.012, .046, .100	2	.019, .031, .038	.009, .056, .104	.007, .053, .109
			$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.012, .051, .101	.013, .048, .100	.008, .057, .087	.25	.007, .057, .096	.007, .045, .099	.012, .049, .098
1	.011, .042, .089	.013, .043, .095	.005, .058, .109	1	.011, .061, .097	.009, .051, .111	.015, .054, .093
2	.031, .044, .085	.012, .044, .099	.011, .051, .097	2	.016, .034, .069	.011, .043, .099	.008, .054, .107
			$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.013, .050, .094	.011, .043, .103	.010, .044, .093	.25	.013, .049, .093	.012, .056, .096	.011, .049, .099
1	.013, .049, .095	.014, .040, .094	.007, .044, .099	1	.020, .029, .045	.012, .044, .093	.007, .053, .102
2	.006, .039, .086	.006, .045, .093	.012, .053, .096	2	.006, .029, .032	.006, .047, .092	.012, .050, .102

	$n = 500$				$n = 1000$		
			$(Y_0, Y_1, X, U) \sim$ **Normal**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .050, .102	.005, .043, .105	.008, .053, .112	.25	.007, .048, .096	.011, .051, .099	.016, .041, .088
1	.011, .030, .068	.010, .050, .103	.010, .050, .100	1	.018, .036, .070	.015, .045, .093	.007, .048, .102
2	.014, .019, .041	.008, .056, .106	.011, .038, .092	2	.016, .031, .043	.010, .050, .102	.006, .045, .098
			$(Y_0, Y_1, X, U) \sim$ **Laplace**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.031, .051, .101	.010, .050, .102	.005, .057, .113	.25	.014, .035, .070	.013, .049, .094	.011, .041, .098
1	.014, .025, .047	.007, .048, .096	.014, .045, .093	1	.012, .019, .024	.008, .054, .103	.013, .046, .098
2	.017, .030, .044	.013, .057, .090	.011, .048, .097	2	.009, .019, .020	.009, .039, .092	.012, .048, .110
			$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .051, .109	.007, .047, .090	.012, .044, .103	.25	.008, .054, .097	.013, .057, .101	.009, .056, .099
1	.014, .040, .103	.010, .052, .104	.012, .046, .093	1	.012, .053, .094	.010, .046, .100	.007, .047, .097
2	.014, .028, .056	.008, .049, .099	.003, .049, .100	2	.014, .034, .060	.012, .048, .096	.006, .046, .092
			$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**				
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.031, .050, .085	.008, .050, .106	.009, .049, .106	.25	.008, .020, .045	.013, .047, .099	.010, .048, .095
1	.021, .056, .063	.007, .052, .100	.008, .050, .093	1	.014, .039, .079	.015, .058, .112	.007, .048, .102
2	.004, .008, .013	.010, .046, .094	.010, .050, .110	2	.006, .010, .015	.006, .052, .095	.009, .046, .095

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^3$ with a zero constant term (hence Z has a symmetric distribution), and $\gamma = [.5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

H.6 Estimated $p(X)$: Case 4 (multivariate X, asymmetric Z, constant used for estimation)

Table H.21. (a) Estimator Properties (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 100, 250$)

		$n = 100$										$n = 250$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**					$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**					$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.001	-.009	.2341	.3906		-.0098	-.0112	.3007	1.255	0	.0038	.0025	.1479	.4496		.0090	.0111	.1784	.6236
TT-BC(Z)	1	-.0024	.0001	.2005	.3611		-.0042	-.0041	.2237	.5539	.4	.0018	.0028	.1292	.4714		.0100	.0047	.1468	.4412
TT(X,$k_n^{(x)}$)	43	.0002	-.0018	.1578	.4694		.0004	-.0035	.2009	.5570	36	.0024	.0025	.1065	.3697		.0032	.0040	.1300	.3322
		$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0018	-.0071	.2505	1.119		.0181	.0047	.2812	.8602	0	-.0077	-.0040	.1682	.7724		.0096	.0003	.2647	3.287
TT-BC(Z)	1	.0044	.0029	.1972	.6031		.0116	.0075	.2053	.3388	.4	-.0057	-.0079	.1361	.5531		.0017	-.0010	.1448	.4179
TT(X,$k_n^{(x)}$)	43	-.0007	.0025	.1639	.4678		.0112	.0126	.1956	.5624	36	-.0036	-.0037	.1127	.4205		.0041	-.0030	.1288	.7524
		$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0116	.0237	.2731	1.060		-.0083	-.0060	.3657	2.378	0	.0026	-.0006	.2626	3.511		.0184	.0048	.3988	4.708
TT-BC(Z)	1	.0077	.0069	.1974	.8383		.0019	.0046	.2106	.3787	.4	-.0004	-.0040	.1322	.5914		.0051	.0064	.1522	.5039
TT(X,$k_n^{(x)}$)	43	.0011	-.0005	.1684	.4809		-.0007	-.0113	.2188	.8537	36	.0031	.0029	.1122	.5632		.0027	.0051	.1378	.4389
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**					$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)					$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0121	.0105	.2309	.7322		-.0168	-.0097	.2759	.7581	0	-.0060	-.0048	.1446	.8858		.0067	.0081	.2260	2.550
TT-BC(Z)	1	.0056	.0054	.2085	.4323		-.0051	-.0007	.2158	.6651	.4	-.0044	-.0057	.1371	.4609		.0007	-.0008	.1391	.6340
TT(X,$k_n^{(x)}$)	43	-.0003	-.0013	.1619	.6084		-.0058	-.0031	.1924	.5170	36	-.0050	-.0039	.1096	.4502		.0090	.0121	.1307	.5142
		$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0006	.0073	.2608	1.472		-.0030	.0021	.2968	1.114	0	.0002	.0024	.1613	.6401		.0120	.0070	.2782	1.545
TT-BC(Z)	1	.0031	.0076	.2018	.6133		.0011	-.0043	.2073	.5586	.4	-.0007	.0016	.1375	.6072		-.0014	.0007	.1738	.6901
TT(X,$k_n^{(x)}$)	43	.0051	.0066	.1625	.7685		-.0011	-.0079	.2027	.8805	36	-.0044	-.0035	.1112	.5162		.0007	.0014	.1465	.7099
		$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)					$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$		Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0027	.0130	.2527	.6184		.0035	-.0022	.3412	2.566	0	.0016	-.0029	.1883	1.297		-.0045	-.0019	.2993	3.542
TT-BC(Z)	1	-.0046	.0017	.1976	.6168		.0003	-.0016	.2043	.7866	.4	.0018	.0027	.1384	.9154		.0017	.0015	.1876	.6710
TT(X,$k_n^{(x)}$)	43	-.0031	-.0001	.1663	.4696		-.0011	-.0007	.1765	.6019	36	-.0022	-.0007	.1138	.4820		-.0009	.0006	.1658	.5662

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.21. (b) Estimator Properties (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 500$, 1000)

$n = 500$

		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0025	.0044	.1035	.4030	-.0001	.0004	.1237	.4991
TT-BC(Z)	.4	.0009	.0010	.0917	.3652	.0033	.0049	.1018	.5364
TT(X,$k_n^{(x)}$)	32	.0018	-.0003	.0787	.5459	.0039	.0049	.0919	.4999
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0044	-.0070	.1335	1.123	.0054	.0087	.1747	2.451
TT-BC(Z)	.4	-.0052	-.0054	.0945	.7193	-.0014	.0004	.1011	.4542
TT(X,$k_n^{(x)}$)	32	-.0024	-.0019	.0826	.6148	.0029	.0037	.0965	.4653
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0038	-.0032	.1430	1.541	.0052	-.0002	.3294	5.613
TT-BC(Z)	.4	-.0025	-.0001	.0903	.4321	-.0033	.0001	.1020	.5952
TT(X,$k_n^{(x)}$)	32	-.0038	-.0033	.0843	.4389	-.0023	-.0024	.1011	.3906

$n = 1000$

		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0009	.0005	.0760	.7358	-.0034	-.0072	.0998	1.901
TT-BC(Z)	.2	-.0002	-.0001	.0655	.3758	-.0025	-.0051	.0729	.7344
TT(X,$k_n^{(x)}$)	29	.0021	.0015	.0540	.3961	-.0021	-.0039	.0668	.5377
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0045	.0065	.1042	1.486	.0016	.0047	.1881	4.400
TT-BC(Z)	.2	.0019	.0027	.0699	.4224	.0033	.0064	.0774	.7633
TT(X,$k_n^{(x)}$)	29	.0022	.0061	.0595	.9541	.0033	.0020	.0709	.8668
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0081	.0040	.1333	2.447	-.0083	-.0039	.2280	4.591
TT-BC(Z)	.2	.0017	.0001	.0713	.7050	-.0039	-.0024	.0767	.6932
TT(X,$k_n^{(x)}$)	29	.0032	.0019	.0625	.5431	-.0009	.0012	.0708	.6173

$n = 500$

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0009	.0025	.1030	.5747	-.0039	-.0029	.1606	1.924
TT-BC(Z)	.4	.0009	.0004	.0959	.4982	-.0027	-.0012	.1018	.6396
TT(X,$k_n^{(x)}$)	32	.0011	.0027	.0802	.6388	.0025	.0049	.0969	.5691
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0022	.0011	.1157	.6546	.0052	.0109	.3400	3.561
TT-BC(Z)	1	.0005	-.0001	.0982	.4631	-.0004	-.0012	.1523	.7109
TT(X,$k_n^{(x)}$)	43	.0019	-.0007	.0808	.5582	.0010	.0010	.1303	.6092
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	-.0019	.0018	.1313	1.169	.0120	.0021	.4552	5.130
TT-BC(Z)	1	-.0021	-.0051	.0962	.5040	.0004	.0014	.1643	.9890
TT(X,$k_n^{(x)}$)	43	-.0018	.0012	.0837	.4986	.0012	.0001	.1346	.8773

$n = 1000$

		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ $(\kappa = 17)$				$\beta = .25$ $(\kappa = 5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0001	-.0027	.0694	.5605	.0022	-.0008	.1351	2.744
TT-BC(Z)	.2	-.0001	-.0024	.0661	.7417	.0019	.0035	.0709	.5282
TT(X,$k_n^{(x)}$)	29	.0002	.0017	.0534	.6806	.0024	-.0002	.0658	.7388
		$\beta = 1$ $(\kappa = 2)$				$\beta = 1$ $(\kappa = 2)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.0024	.0015	.0756	.8773	-.0330	-.0098	.3507	4.652
TT-BC(Z)	.2	.0011	.0011	.0671	.4658	-.0006	-.0012	.1300	.5598
TT(X,$k_n^{(x)}$)	29	.0005	-.0005	.0567	.5478	.0013	.0001	.1048	.6205
		$\beta = 2$ $(\kappa = 1.25)$				$\beta = 2$ $(\kappa = 1.5)$			
Estimator	Tr%	Mean	Med	MSE	KS$_{.05}$	Mean	Med	MSE	KS$_{.05}$
No Trim	0	.005	-.0065	.1045	1.358	.0202	.0091	.3980	6.222
TT-BC(Z)	.2	-.0020	-.0027	.0697	.5956	-.0001	-.0021	.1277	.7721
TT(X,$k_n^{(x)}$)	29	-.0017	-.0023	.0560	.6524	-.0006	.0003	.1031	.6626

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$. KS$_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than KS$_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.22. Rejection Frequencies (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation

$n = 100$					$n = 250$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.013, .047, .098	.011, .050, .099	.010, .049, .103	.25	.013, .054, .102	.010, .056, .096	.012, .049, .093
1	.009, .047, .082	.010, .054, .096	.012, .048, .101	1	.015, .048, .086	.008, .045, .102	.014, .056, .096
2	.017, .053, .090	.014, .058, .106	.014, .052, .104	2	.005, .016, .023	.006, .049, .107	.017, .054, .093
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.018, .041, .067	.006, .053, .107	.011, .060, .108	.25	.016, .050, .084	.013, .051, .095	.010, .054, .107
1	.020, .052, .085	.009, .047, .106	.009, .054, .104	1	.013, .018, .037	.006, .052, .102	.009, .044, .088
2	.015, .034, .056	.010, .056, .105	.018, .049, .094	2	.008, .019, .026	.012, .052, .101	.007, .058, .109
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .049, .101	.008, .050, .100	.016, .048, .096	.25	.011, .051, .100	.010, .052, .095	.008, .050, .112
1	.014, .045, .070	.005, .037, .091	.015, .049, .091	1	.016, .052, .086	.010, .048, .097	.011, .058, .095
2	.014, .050, .092	.004, .048, .106	.013, .046, .093	2	.019, .053, .084	.010, .055, .108	.009, .049, .108
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .052, .094	.012, .060, .102	.010, .051, .103	.25	.010, .020, .037	.010, .046, .094	.008, .052, .107
1	.012, .041, .070	.006, .050, .103	.031, .041, .092	1	.006, .035, .039	.012, .048, .095	.013, .061, .112
2	.005, .032, .056	.014, .055, .107	.008, .046, .094	2	.002, .004, .0005	.006, .045, .095	.006, .047, .096

$n = 500$					$n = 1000$		
$(Y_0, Y_1, X, U) \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.031, .054, .089	.015, .057, .090	.013, .059, .101	.25	.014, .035, .075	.009, .054, .101	.010, .052, .104
1	.019, .048, .091	.006, .042, .097	.006, .044, .099	1	.020, .042, .068	.009, .050, .105	.014, .046, .095
2	.021, .053, .078	.009, .056, .101	.013, .047, .094	2	.019, .042, .061	.007, .045, .096	.014, .042, .095
$(Y_0, Y_1, X, U) \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.011, .042, .098	.006, .050, .102	.009, .049, .108	.25	.011, .039, .076	.010, .054, .101	.018, .052, .092
1	.015, .035, .052	.008, .052, .099	.009, .051, .101	1	.009, .011, .020	.007, .049, .096	.013, .053, .105
2	.013, .016, .025	.006, .055, .104	.011, .050, .093	2	.013, .019, .030	.008, .061, .106	.010, .048, .112
$(Y_0, Y_1, X) \sim$ **Normal**, $U \sim$ **Laplace**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.010, .056, .101	.005, .055, .101	.011, .048, .102	.25	.009, .041, .102	.005, .041, .101	.007, .045, .097
1	.019, .041, .080	.011, .054, .102	.009, .044, .101	1	.017, .047, .091	.012, .049, .094	.009, .046, .101
2	.010, .042, .080	.008, .051, .104	.012, .051, .096	2	.012, .038, .068	.007, .047, .103	.010, .060, .104
$(Y_0, Y_1, X) \sim$ **Laplace**, $U \sim$ **Normal**							
β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)	β	No Trim	TT–BC(Z)	TT(X,$k_n^{(x)}$)
.25	.012, .029, .050	.006, .046, .104	.005, .053, .100	.25	.015, .030, .044	.012, .043, .094	.011, .045, .089
1	.003, .021, .031	.005, .046, .096	.011, .048, .093	1	.005, .029, .030	.007, .046, .096	.006, .045, .097
2	.001, .001, .002	.011, .054, .105	.013, .054, .110	2	.002, .002, .003	.013, .054, .098	.008, .046, .105

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$; "TT(Z)" is the tail-trimmed estimator $\hat{\theta}_n^{(tz)}$ and "TT–BC(Z)" is the bias-corrected tail-trimmed $\hat{\theta}_n^{(tz:o)}$: both use *sample mean-centering* for trimming. "TT(X)" is $\theta_n^{(tx)}$; and "TT(X,k)" is the adaptive version $\hat{\theta}_n^{(tx)}$ of $\theta_n^{(tx)}$.

Table H.23. (a) Trim-by-$p(X)$ Estimator Properties (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 100, 250$)

	$n = 100$									$n = 250$									
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.008	-.0007	.2338	.4173	-.0089	-.0020	.2783	1.187	0	-.0037	-.0018	.1595	1.412	.0036	-.0044	.1872	1.302
2n/ln(n)	44	.0013	.0002	.1575	.5864	.0064	.0095	.1980	.5411	36	.0009	-.0007	.1053	.5547	-.0002	.0010	.1278	.3678
4n/ln(n)	86	-.0037	-.0036	.0759	.7852	-.0050	-.0084	.1029	.7683	72.8	.0032	-.0004	.0667	.7887	-.0017	-.0031	.0894	.3602
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0143	.0078	.2724	1.233	-.0026	-.0116	.4328	2.234		.0095	.0086	.1882	1.487	-.0164	-.0065	.2756	3.318
2n/ln(n)	44	-.0002	-.0008	.1649	.5350	.0063	.0093	.2114	.6215		.0005	-.0017	.1058	.5077	.0023	-.0006	.1415	.4605
4n/ln(n)	86	-.0015	.0020	.0821	1.021	-0018	-.0033	.1093	.7616		-.0015	-.0021	.0698	.7710	-.0016	.0002	.0958	.5356
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0138	-.0131	.2666	.8979	.0019	-0039	.3249	1.528	0	-.0169	-.0032	.3323	4.291	-.0186	-.0002	.4583	5.540
2n/ln(n)	44	-.0023	-.0030	.1811	.5568	.0122	.0122	.2460	.9700	36	-.0064	-.0014	.1262	.5995	.0011	.0054	.1584	.9387
4n/ln(n)	86	-.0015	-.0029	.0841	.7258	.0013	-.0040	.1205	.6890	72.8	-.0013	-.0031	.0751	.4829	-.0047	-.0064	.1019	.4870
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0028	-.0018	.2290	.6309	-.0020	-.0073	.2725	.7330	0	-.0021	-.0026	.1404	.6227	.0026	-.0095	.1765	.8684
2n/ln(n)	44	.0012	-.0030	.1610	.6157	-.0085	-.0038	.1961	.5751	36	.0014	.0055	.1052	.5301	.0031	.0059	.1255	.4861
4n/ln(n)	86	-.0041	-.0035	.0767	.3392	.0052	.0066	.1077	.4095	72.8	-.0003	-.0015	.0674	.5029	.0038	.0017	.0901	.4904
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0021	.0028	.2257	.9290	-.0132	-.0177	.3145	2.658	0	-.0064	-.0111	.1646	.7817	.0231	.0167	.4412	3.331
2n/ln(n)	44	-.0016	-.0050	.1574	.4061	-.0078	-.0032	.1821	1.198	36	-.0017	-.0029	.1158	.5443	-.0078	-.0045	.2012	1.192
4n/ln(n)	86	.0016	-.0001	.0807	.5945	-.0016	-.0022	.0782	.7682	72.8	.0005	.0019	.0708	.6226	.0008	-.0005	.1211	.7892
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0118	-.0157	.2694	1.373	-.0542	-.0072	.4252	5.493	0	-.0117	-.0103	.1953	1.157	-.0062	-.0102	.4222	6.192
2n/ln(n)	44	.0007	-.0027	.1722	.6076	.0054	.0042	.2043	1.320	36	-.0067	-.0088	.1143	.5215	.0101	.0005	.2291	1.542
4n/ln(n)	86	-.0013	-.0063	.0866	.7233	.0031	.0031	.1121	.8976	72.8	.0046	.0010	.0746	.6978	-.0012	-.0007	.0687	.6767

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. . A constant term is used for estimation. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.23. (b) Trim-by-$p(X)$ Estimator Properties (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation, $n = 500, 1000$)

		$n = 500$								$n = 1000$								
		$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**					$(Y_0, Y_1, X, U) \sim$ **Normal**				$(Y_0, Y_1, X, U) \sim$ **Laplace**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0007	-.0025	.1078	.5176	.0063	.0047	.1206	.5939	0	.0006	.0015	.0773	.7571	.0006	-.0007	.0957	1.209
2n/ln(n)	32	-.0021	-.0033	.0783	.8156	.0061	.0086	.0940	.7399	29	.0013	-.0005	.0569	.4959	-.0014	.0006	.0635	.5501
4n/ln(n)	64.4	-.0011	-.0017	.0559	.4471	-.0004	.0011	.0731	.5725	59	.0005	-.0010	.0427	.6031	-.0017	-.0015	.0520	.4311
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0030	.0033	.1379	1.856	-.0034	-.0065	.2366	4.055	0	-.0022	.0018	.0995	1.672	.0022	-.0017	.1783	4.177
2n/ln(n)	32	.0032	.0038	.0808	.4370	.0002	.0030	.1018	.6480	29	.0010	.0031	.0562	.6394	-.0018	-.0027	.0711	.4644
4n/ln(n)	64.4	-.0001	-.0017	.0566	.6303	-.0004	-.0018	.0763	.7659	59	.0027	.0045	.0451	.6949	-.0023	-.0032	.0566	.4911
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0074	-.0067	.2166	3.274	-.0029	.0057	.3672	5.442	0	-.0135	.0005	.3187	7.026	-.0228	-.0058	.7105	8.622
2n/ln(n)	32	.0033	-.0019	.0944	.6073	.0017	.0096	.1179	.8333	29	-.0006	-.0044	.0670	.7328	-.0038	-.0041	.0884	.7481
4n/ln(n)	64.4	.0015	.0012	.0610	.4142	.0029	.0034	.0806	.5551	59	-.0015	-.0027	.0462	.7627	-.0006	.0014	.0604	.7905
		$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**					$(Y_0, Y_1, X) \sim$ **Norm,** $U \sim$ **Lap**				$(Y_0, Y_1, X) \sim$ **Lap,** $U \sim$ **Norm**			
		$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)					$\beta = .25$ ($\kappa = 17$)				$\beta = .25$ ($\kappa = 5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	.0049	.0049	.0989	.5523	-.0024	.0028	.1394	.8916	0	-.0009	-.0033	.0716	.5942	.0083	.0083	.1024	1.479
2n/ln(n)	32	.0032	.0060	.0795	.4580	-.0038	-.0001	.0929	.6313	29	.0005	-.0050	.0555	.6320	.0018	-.0004	.0646	.4209
4n/ln(n)	64.4	.0006	.0002	.0548	.5141	-.0020	-.0043	.0705	.7627	59	.0019	.0014	.0421	.4872	-.0012	-.0028	.0528	.6083
		$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)					$\beta = 1$ ($\kappa = 2$)				$\beta = 1$ ($\kappa = 2$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0065	-.0034	.1133	.4272	.0331	.0256	.4623	5.552	0	-.0040	-.0072	.0770	.7379	.0056	.0203	.4787	7.892
2n/ln(n)	32	-.0030	-.0046	.0815	.5358	-.0141	-.0054	.2042	1.192	29	-.0021	-.0024	.0554	.5443	.0016	.0014	.2300	1.209
4n/ln(n)	64.4	-.0030	-.0040	.0549	.4060	-.0071	-.0009	.1044	.9543	59	.0019	.0005	.0412	.6090	.0010	.0002	.1002	.5672
		$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)					$\beta = 2$ ($\kappa = 1.25$)				$\beta = 2$ ($\kappa = 1.5$)			
Estimator	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$	Tr%	Mean	Med	MSE	$KS_{.05}$	Mean	Med	MSE	$KS_{.05}$
No Trim	0	-.0072	-.0020	.1500	1.679	-.0067	-.0141	.4409	6.551	0	-.0011	-.0067	.1084	1.849	-.0240	-.0054	.4881	8.792
2n/ln(n)	32	-.0007	-.0029	.0850	.5598	.0011	-.0006	.2031	1.454	29	.0017	.0042	.0644	.5552	.0009	-.0008	.2341	1.540
4n/ln(n)	64.4	-.0011	-.0040	.0602	.6853	.0008	-.0020	.1122	1.043	59	.0017	.0014	.0461	.3477	.0001	.0001	.0975	.5742

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. “No Trim” is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations. $KS_{.05}$ is the Kolmogorov-Smirnov test statistic divided by its 5% critical value: values above 1 indicate rejection of standard normality at the 5% level. Tr% is the percent of observations Z_i trimmed. κ is the tail index of $Z = h(X)Y$. Other than $KS_{.05}$, all values are averages over the randomly drawn 10,000 samples.

Table H.24. Trim-by-$p(X)$ Rejection Frequencies (Case 4: Asymmetric Z, Estimated $p(X)$, X is multivariate, a constant term used in estimation

$n=100$				$n=250$			
$(Y_0,Y_1,X,U)\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.013, .047, .099	.006, .054, .103	.012, .055, .100	.25	.011, .049, .092	.009, .054, .096	.010, .046, .095
1	.010, .047, .082	.009, .054, .107	.017, .047, .092	1	.013, .026, .061	.010, .056, .095	.014, .053, .098
2	.012, .047, .093	.012, .047, .094	.012, .066, .098	2	.007, .014, .018	.012, .058, .107	.012, .046, .099
$(Y_0,Y_1,X,U)\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.012, .047, .090	.010, .055, .094	.014, .060, .095	.25	.012, .039, .072	.012, .055, .097	.011, .046, .094
1	.017, .048, .060	.009, .051, .111	.017, .054, .087	1	.018, .028, .046	.010, .045, .100	.012, .052, .093
2	.020, .044, .066	.011, .051, .095	.016, .062, .104	2	.011, .013, .016	.019, .053, .100	.010, .048, .109
$(Y_0,Y_1,X)\sim$ **Normal**, $U\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.012, .048, .107	.015, .057, .103	.015, .050, .098	.25	.017, .049, .099	.011, .051, .101	.010, .051, .096
1	.019, .039, .084	.008, .048, .117	.009, .051, .105	1	.017, .047, .096	.010, .052, .101	.012, .055, .105
2	.019, .041, .079	.011, .047, .101	.015, .050, .086	2	.015, .036, .066	.008, .051, .098	.007, .049, .099
$(Y_0,Y_1,X)\sim$ **Laplace**, $U\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.011, .048, .102	.011, .041, .101	.013, .047, .096	.25	.015, .042, .095	.008, .051, .106	.010, .060, .099
1	.003, .020, .062	.013, .047, .094	.014, .054, .109	1	.012, .029, .034	.012, .049, .108	.006, .046, .097
2	.012, .023, .041	.007, .045, .092	.006, .053, .112	2	.007, .007, .009	.009, .052, .110	.008, .046, .105

$n=500$				$n=1000$			
$(Y_0,Y_1,X,U)\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.010, .052, .109	.008, .042, .103	.014, .042, .099	.25	.011, .044, .083	.008, .052, .098	.011, .046, .093
1	.013, .030, .053	.010, .046, .103	.006, .059, .104	1	.021, .039, .068	.011, .046, .091	.006, .053, .105
2	.017, .025, .032	.008, .052, .115	.006, .043, .098	2	.004, .004, .006	.015, .049, .099	.011, .052, .097
$(Y_0,Y_1,X,U)\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.016, .046, .092	.010, .041, .101	.012, .049, .097	.25	.014, .041, .074	.012, .052, .100	.011, .052, .094
1	.010, .017, .023	.013, .051, .104	.006, .043, .101	1	.014, .020, .032	.009, .054, .099	.013, .051, .094
2	.012, .019, .031	.008, .060, .102	.012, .052, .100	2	.003, .004, .004	.008, .044, .092	.010, .043, .093
$(Y_0,Y_1,X)\sim$ **Normal**, $U\sim$ **Laplace**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.012, .051, .091	.014, .050, .091	.007, .046, .097	.25	.007, .054, .103	.010, .059, .104	.008, .051, .105
1	.009, .043, .097	.006, .049, .097	.013, .045, .098	1	.014, .042, .090	.006, .044, .099	.009, .050, .092
2	.023, .036, .059	.010, .052, .097	.008, .052, .093	2	.011, .035, .062	.012, .047, .103	.011, .046, .101
$(Y_0,Y_1,X)\sim$ **Laplace**, $U\sim$ **Normal**							
β	No Trim	2n/ln(n)	4n/ln(n)	β	No Trim	2n/ln(n)	4n/ln(n)
.25	.015, .045, .078	.013, .048, .098	.006, .041, .094	.25	.017, .041, .067	.014, .046, .088	.012, .046, .094
1	.006, .026, .036	.008, .045, .092	.008, .046, .096	1	.007, .035, .041	.012, .045, .087	.010, .052, .097
2	.005, .006, .009	.007, .039, .089	.012, .052, .097	2	.001, .001, .003	.007, .045, .092	.012, .051, .100

The treatment assignment is $D = I(\gamma' X > U)$, where $X = [X_i]_{i=1}^4$ with a constant term .25 (hence Z has an asymmetric distribution), and $\gamma = [.25, .5, \beta, \beta/2]$. The propensity score $p(X)$ is estimated by maximum likelihood in order to compute Z. A constant term is used for estimation. Values are rejection frequencies of the null hypothesis ATE = 0, at the 1%, 5%, 10% levels. "No Trim" is the untrimmed estimator $\tilde{\theta}_n$. The values $\{2n/\ln(n), 4n/\ln(n)\}$ are the total number of trim-by-$p(X)$ observations.

References