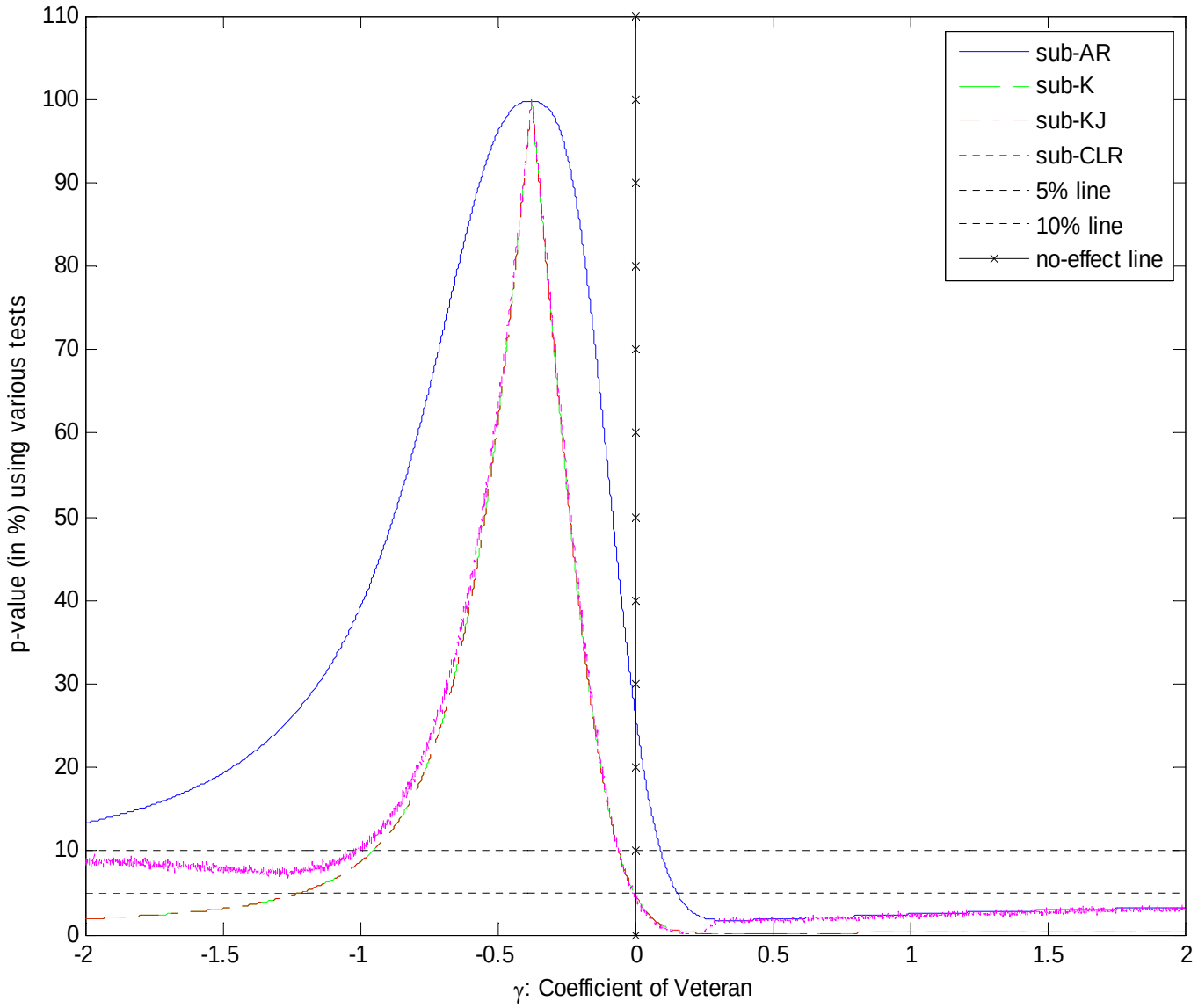

Figure 1: p-values in % plotted for the various plug-in based weak identification (robust) tests for the coefficient $\gamma$of Veteran Status. The dotted horizontal lines represent 10% and 5% respectively, meaning that values of γ for which the p-value of the subset-AR test is above these lines cannot be rejected by the subset-AR test at the 10% and 5% level respectively. It was believed that the same holds for the subset-K, subset-KJ and subset-CLR tests. Accordingly, all non-negative values of γ (to the right of the vertical no-effect line) can be rejected at the 5% level by the subset-K, subset-KJ and subset-CLR tests.

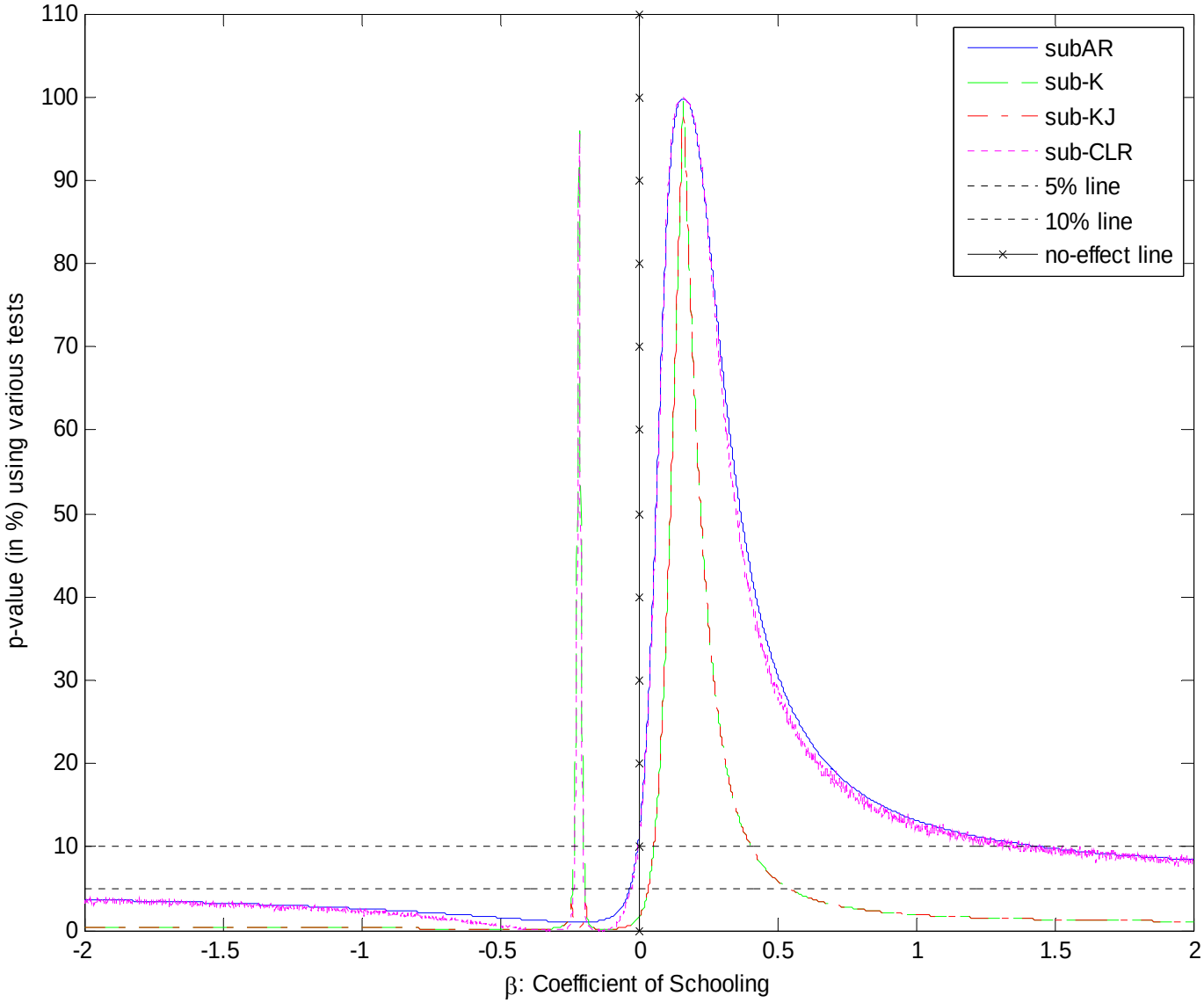

Figure 2: p-values in % plotted for the various plug-in based weak identification (robust) tests for the coefficient β of years of Schooling. The dotted horizontal lines represent 10% and 5% respectively, meaning that values of β for which the p-value of the subset-AR test is above these lines cannot be rejected by the subset-AR test at the 10% and 5% level respectively. It was believed that the same holds for the subset-K, subset-KJ and subset-CLR tests. The subset-K test is known to have spikes in the p-value at spurious points (around -.2, to the left of the vertical no-effect line). The subset-KJ test can sometimes solve this problem, but it does not happen in our application. Negative values of β are rejected at the 10% level according to the subset-AR and subset-CLR tests.

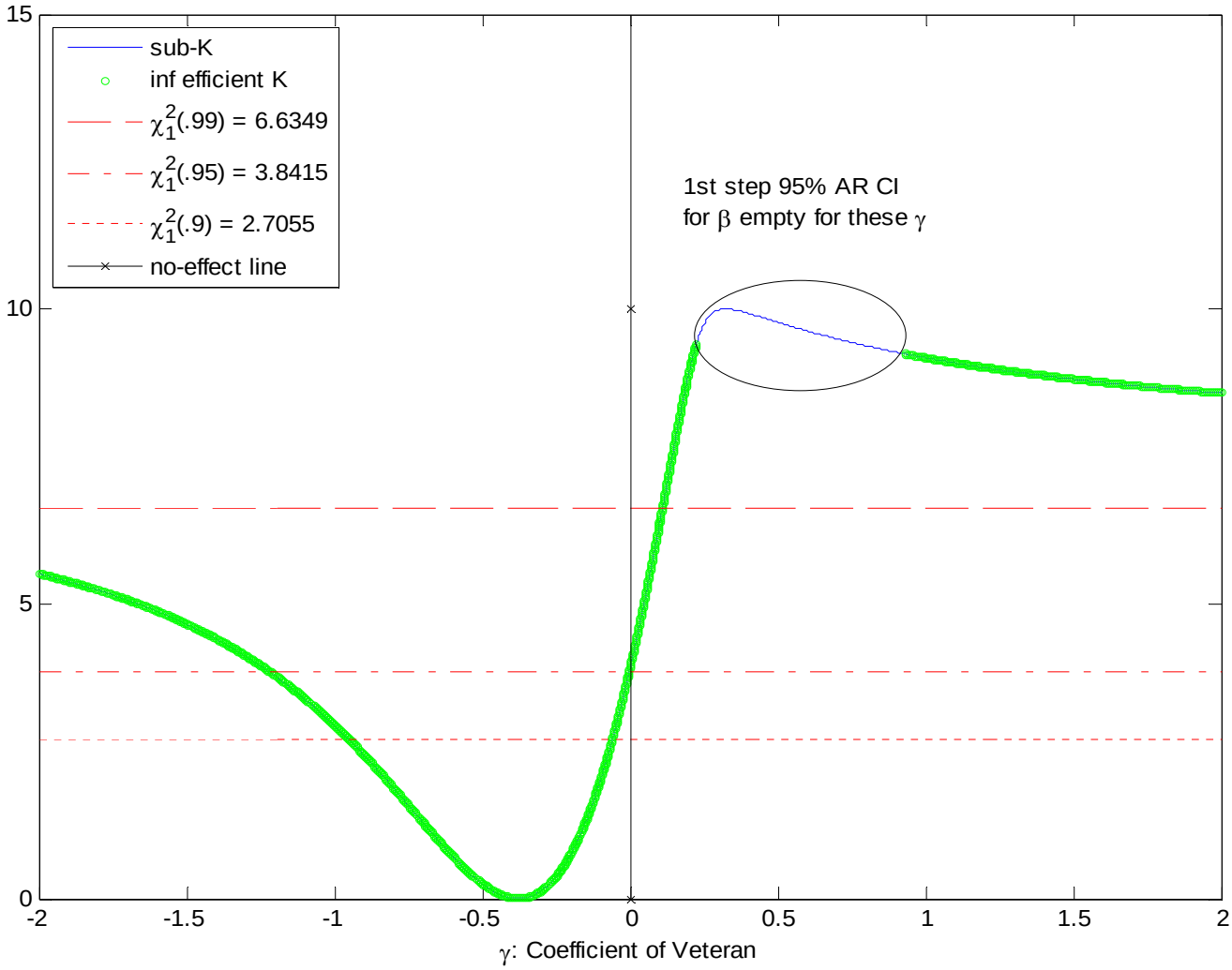

Figure 3: Plotted are the subset-K statistic and the projection statistic for various values of the coefficient $\gamma$of Veteran Status. The dotted horizontal lines represent 99%, 95% and 90% quantile of a central chi-squared distribution with one degree of freedom. The projection statistic for a given value of γ is the infimum of the efficient K statistic where the infimum is taken over a 95% confidence region for the nuisance parameter β (for a given value of γ). This 95% confidence interval is obtained by inverting the AR test. The infimum is defined as plus infinity if the said confidence interval is empty (see to the right of the vertical no-effect line). In our context, the projection test is used to support the rejection of values of γ by the subset-K test by putting non-trivial upper bounds to the levels of rejection that are at most 5% more than those believed to the case with the subset-K test. This happens because the projection statistic (when not plus infinity) is very close to the subset-K statistic.

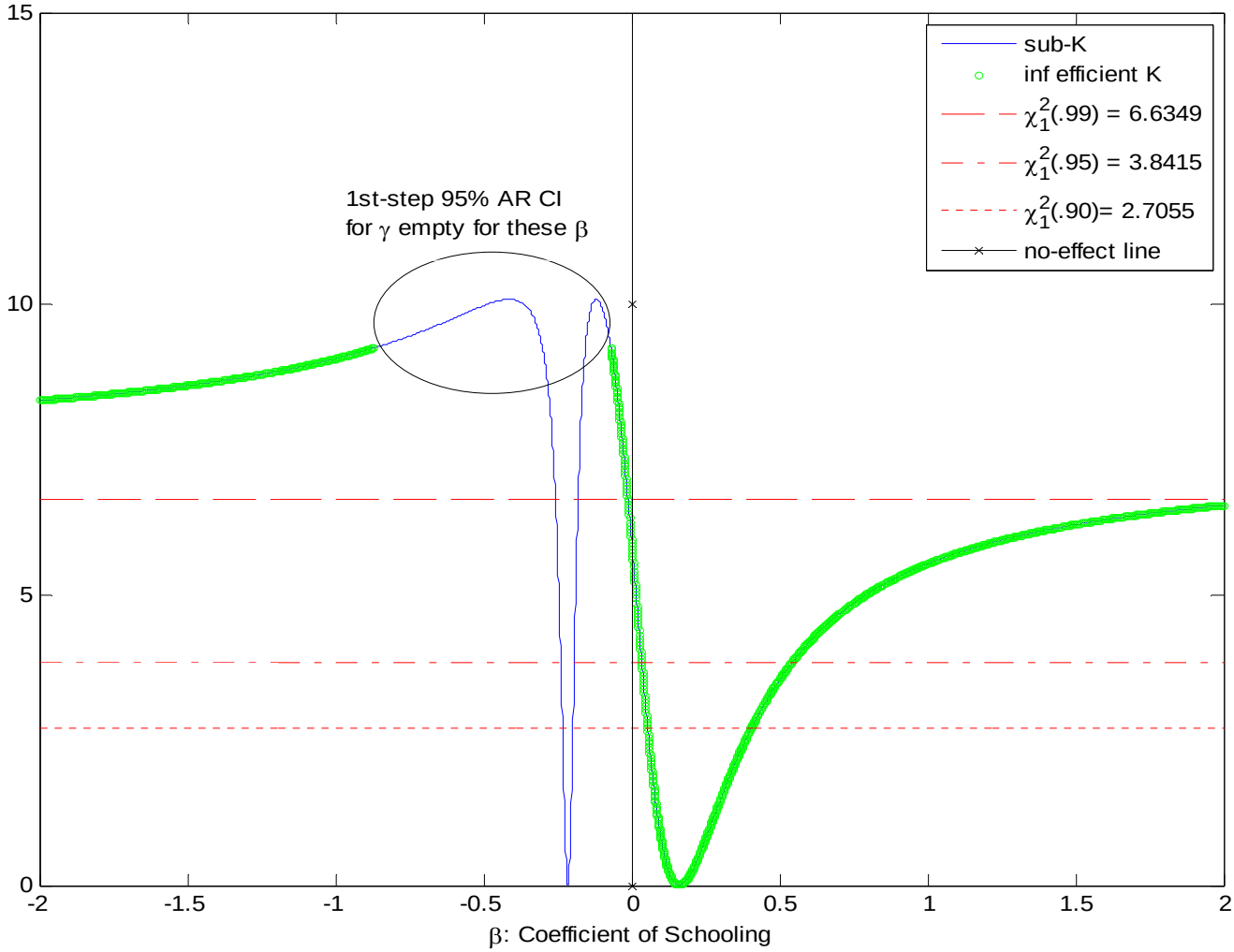

Figure 4: Plotted are the subset-K statistic and the projection statistic for various values of the coefficient β of years of Schooling. The dotted horizontal lines represent 99%, 95% and 90% quantile of a central chi-squared distribution with one degree of freedom. The projection statistic for a given value of β is the infimum of the efficient K statistic where the infimum is taken over a 95% confidence region for the nuisance parameter γ (for a given value of β). This 95% confidence interval is obtained by inverting the AR test. The infimum is defined as plus infinity if the said confidence interval is empty (see to the left of the vertical no-effect line). In our context, the projection test is used to support the rejection of values of γ by the subset-K test by putting non-trivial upper bounds to the levels of rejection that are at most 5% more than those believed to the case with the subset-K test. This happens because the projection statistic (when not plus infinity) is very close to the subset-K statistic. In addition, the projection statistic also helps to rule out values to the left of the no-effect line that could not be rejected by the subset-K test due to the spurious dip in value of the latter statistic at those points (see Figure 2).